A Study of Self-reciprocal Polynomials

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Let p be a prime and $\mathbb{F}_p$ be the finite field with p elements. A *self-reciprocal polynomial* is a polynomial whose coefficients form a palindrome. For example, $f(x) = x^6 + 2x^4 + 2x^2 + 1$.

The nth Dickson polynomial of the first kind is defined by For $a \in \mathbb{F}_p$,

$$D_n(x, a) = \sum_{i=0}^{\lfloor n/2 \rfloor} \frac{n}{n-i} \binom{n-i}{i} (-a)^i \, x^{n-2i}.$$

We are primarily interested in the following question: When is $D_n(x, a)$ a self-reciprocal polynomial over $\mathbb{F}_p$? In this talk, we will explain the self-reciprocal behavior of Dickson polynomials of the first kind over $\mathbb{F}_p$. We will also give a classification of self-reciprocal polynomials arising from Dickson polynomials of an arbitrary kind over $\mathbb{F}_p$.

Moreover, we will explain the applications of self-reciprocal polynomials in the area of coding theory, both in cyclic codes and in error-correcting codes.