

Jennifer and Andrew,

Both your presentation and the final project paper were excellent. You did a really good job making the  $F_4$  algorithm understandable for the class. Your paper also presents everything very well. I have a few relatively minor comments for you:

1. You say that “Buchberger’s Algorithm is the most standard method of computing Gröbner bases in most computer algebra systems such as Maple.” While that was true in the past, the current “state of the art” is actually much more interesting. In fact, as you describe at the end of the paper, the  $F_4$  algorithm itself is a big part of Maple’s Gröbner basis routine in the latest versions. There is a discussion on pages 589-590 of all the different Gröbner basis algorithms that are incorporated into the `Basis` command that we used all the time and there are actually even two different implementations of  $F_4$  that are a part of that. Maple selects an approach to use based on the number of variables, how complicated the polynomials are, what monomial order is specified, and so forth. This is all transparent to the user, but as you saw, it is possible to set the `infolevel` option to see what’s going on “under the hood” if you want to. A number of the different components of the Gröbner `Basis` command in Maple are things we did not talk about, but that are connected to some of the ideas we were seeing in the last lab and problem set (the FGLM procedure in particular).
2. A writing point: It’s OK to use first person in mathematical writing, especially if you want to “bring the reader into” what you are saying. That way you can also avoid passive constructions like the one you have here. It sounds much better to say “we will outline the basic form of the  $F_4$  algorithm” than “outlined will be the basic form of the  $F_4$  algorithm.”
3. It would have been much better to put the pseudocode first, and then explain what’s going on. I find this sort of written out form of the algorithm is not easy to follow without having the pseudocode to look at. The same goes for the later discussion of the `ComputeM` procedure.
4. What you are saying about  $F_4$  reducing to Buchberger if  $B$  consists of a single pair is essentially true. But I think there are actually cases where elements of the ideal corresponding to rows of the reduced echelon form matrix would turn out to be different from the polynomials produced by the division algorithm computation of  $\overline{S(f_i, f_j)}^G$ . The reason is that the row-reduction of the matrix produced by `ComputeM` is not restricted in the same way that choice of divisors in the division algorithm is restricted.
5. Give reference to Theorem 2 in Section 10.3 of IVA.
6. The `LinearAlgebra` package in Maple also has a row-reduced echelon form command called `ReducedRowEchelonForm`. Is there a reason you didn’t want to use that? The `MTM` package is really designed to let you combine Maple and MATLAB.

Final Project Presentation: 98 (A)

Final Project Paper 96 (A)