

Jack,

Your final project presentation and paper were both very good. (The presentation in class was somewhat better, in fact, than your Honors presentation. What happened there? It seemed as though you might not have practiced enough, or perhaps not looked over things again before giving the Honors presentation.) Your LaTeX proficiency is impressive, but I noted a few small points on the hardcopy, for future reference.

Specific Comments

0. The fact that, for all i , $\text{multideg}(A_i g_i)$ is never larger than $\text{multideg}(f)$ when you use the division algorithm to write

$$f = A_1 g_1 + \cdots + A_t g_t + r$$

is a consequence of how the division algorithm works. Look back at Chapter 2, Section 3, Theorem 3 and the proof. This means that expressions computed by division by a Gröbner basis for I for elements $f \in I$ are always standard representations. (You say this later.)

1. I think it's more accurate to say that Buchberger's Criterion is used as the *correctness criterion* in Buchberger's Algorithm. It's not the *means* for computing Gröbner bases; it's how you tell that, when the algorithm stops computing S -polynomial remainders, the set of polynomials you have is a Gröbner basis for the ideal generated by the input polynomials.
2. This is not quite accurate. First see point 0. above. This means that expressions computed by the division algorithm are always standard representations, as shown in Chapter 2, Section 3, Theorem 3. The issues you need to appreciate are:
 - (a) There can also be standard representations of f that are *different from* the one computed by the division algorithm in some cases, and
 - (b) Not all representations of a given f as

$$f = A_1 g_1 + A_2 g_2 + \cdots + A_{t-1} g_{t-1} + A_t g_t$$

are standard. For instance, if B is an arbitrary polynomial, then

$$f = (A_1 + B g_t) g_1 + A_2 g_2 + \cdots + A_{t-1} g_{t-1} + (A_t - B g_1) g_t$$

is also true, because the $B g_t g_1$ and $-B g_1 g_t$ terms cancel. But this other representation for f can easily fail to be a standard representation if the leading term of B is sufficiently large in the monomial order. This is what you were trying to say, I think, but it did not quite make it to the page correctly.

3. You should always list the references you used for a paper like this (a research paper) even if there was only one.

Final Project Presentation: 96 (A)

Final Project Paper: 96 (A)