SIMU 1999–Gröbner Bases Seminar
Week 1 Computer Laboratory Exercises

Background and Goals

A large part of our work this summer will be carrying out calculations of Gröbner bases and other computations within the computer algebra system called *Maple*. This week, we will want to get familiar doing mathematics in this context.

General Information about Maple

Launching Maple

To get into Maple, you will need to:

1) Turn the computer and monitor on.
2) Wait a few minutes while the computer "boots up" and launches Windows 95. Eventually you will see a prompt to log onto the Novell campus network. It is possible to *bypass that*, by pressing ENTER. IMPORTANT NOTE: To get paper printouts of your work, though, you will need to log onto the network. See printing directions at the end of this handout.
3) You should now see the Windows 95 desktop with "icons" (technically, "shortcuts" in Windows 95) for Maple, etc. You will double click on the Maple V Release 5 icon – move the cursor arrow with the mouse until it is over the `Maple V 5` icon and quickly click the left mouse button *twice* to launch Maple.
4) After a few more seconds you should see a new Maple V Release 5 window, with a "subwindow" marked Untitled (1) opened inside it. The Untitled (1) window is a blank *Maple worksheet*.

Maple Worksheets

Worksheets are integrated text/graphics/mathematics documents where any or all of the following can be done:

a) you can type in commands from the keyboard to ask Maple to perform many different kinds of calculations, read data from external files, store work in files, etc.
b) output generated by Maple from your input commands (numerical values, symbolic formulas, and graphics) will be displayed,
c) you can modify commands, generating new output, store your results for use later, etc.
d) you can enter text to annotate and explain the results of computations.

Much of your interactive "management" of a worksheet in progress will take place through the File, Edit, View, Insert, etc. pull-down menus across the top, the "tool bar" below them with the icons for various operations (can you guess what each of them does?) the "scroll bar" on the right that you can use to move around within the worksheet, to see previous input and output lines, etc. Most of the common operations appear both as items in one of the pull-down menus and as toolbar buttons. The toolbar buttons are faster to use, but somewhat limited in the options they offer, so you will want to have a general idea what is accessible from where in the pull-down menus.

Input and Text Regions in a Worksheet

A new worksheet window will be labeled something like Untitled (1) window. On the first line of the worksheet, there is a [> in red. This is the Maple *input prompt* – the signal that Maple is ready to accept a command from you and try to execute it. A new input prompt will be generated automatically in a worksheet each time you enter a command and execute it *at the end of a worksheet in progress*. You can also go back and insert new commands and output at any point in a worksheet as follows. You can *insert an input prompt* and an input region at any point in the worksheet by placing the cursor at the desired location and pressing the toolbar button marked with [>. (After executing an inserted command like this, Maple will drop to the next input line, so to insert several input lines in the middle of a worksheet, you will need repeat the above.)

As we mentioned before, Maple worksheets can contain text as well as commands and output. To create a text region you can press the [> button above, then the button marked by a capital T (this changes the region into a text region). Any text can now be entered, and this will be treated as an "inert" comment. That is, it will appear as you enter it when you print out the worksheet, and it will not be treated as Maple input. After you create a text region, if you press the capital sigma (Σ) toolbar button with the cursor placed on that region, you can type in a Maple expression in input format, and have it displayed (in "inert" form) in usual mathematical notation (with raised exponents, no asterisks for multiplication, etc. This is sometimes useful for creating "pretty" worksheets!

Saving and Reloading your Maple Worksheets

When you begin working on a worksheet, you will usually want to *save your work* every once in a while in case a computer problem develops, or in case you need more than one lab session to complete the work you are doing. This can be done most directly by saving to your network drive. (You can also save to a 3 1/2 inch IBM format diskette.) following these directions:

1) If you are using a diskette, insert it in the diskette drive (it's usually called the a: drive). Otherwise go directly to step 2.
2) In Maple, select the SAVE option from the FILE pull-down menu.
3) If you are saving your work for the first time, you will see a SAVE AS dialog box. In the Drives box, highlight the A: or network drive as appropriate, then go to the File Name box, and type in a name for the file where the worksheet will be stored (any string of letters and digits – no spaces – no more than 8 characters long is OK) type in a period after the file name you chose and then the "extension" *mws* for *Maple Work Sheet*. For instance, a good choice might be something like *lab1.mws*. Then click the

OK button to write the worksheet to the network drive or to your diskette. (You will only need to type the filename in once in a session. Subsequent saves just update the file.)

4) When you have the worksheet saved as you want it, you can remove the diskette and exit Maple.
5) To update the worksheet further in a later session, you can read the worksheet back into Maple using the OPEN option from the FILE Menu, or the "open" toolbar button. Get into Maple as above, (if the worksheet is stored on diskette, insert it in the a: drive), and select OPEN from the FILE Menu. Maple will prompt you as above for the name of the worksheet with a dialog box very like the SAVE AS box described above. Highlight the A: or network drive as appropriate, then the worksheet you want, and click OK. IMPORTANT NOTE: When you read a previously created worksheet into a new Maple session, you *have not* actually executed any of the commands in it in the new session. If you want to make use of any results in that worksheet, you will need to execute the commands again. In the EDIT pull-down menu, you will see an EXECUTE option that lets you recompute an entire worksheet or a section of one.

Printing

With the worksheet you want to print open and highlighted, and press the toolbar icon that looks like a printer, and press OK on the PRINT dialog box (the settings should be set up correctly to print automatically). Your output should appear shortly on the printer.

Getting Out

When you leave, quit the Maple window (FILE/EXIT) or the X button on the top right of the window.

Day 1: A First Sample Maple Session

Let's get right down to work and walk through a first sample sample session! First, you will need to get into Windows and Maple as described in the General Information section above.

2D Plotting

The basic Maple command for 2D plotting, used for graphs of the form $y = f(x)$ and for parametric curves $\alpha(t) = (x(t), y(t))$ is called `plot`. The basic format for graphs $y = f(x)$ is

```
plot(function,range,options);
```

where

1) `function` is the function to be plotted – the simplest way to specify one is via a formula for $f(x)$ (an *expression* in Maple)
2) `range` is the range of x-values you want to see plotted, entered in the form `x = low..high`, and
3) `options` can be used to control the form of the plot if desired. No options need be specified however if you don't want to. More on this later.

Example 1

To plot $y = x^4 - 2x^3 + x - 5\sin(x^2)$ for $-2 \le x \le 1$, you could use the Maple `plot` command with no options:

```
plot(x^4-2*x^3+x-5*sin(x^2),x=-2..1);
```

Try this now. (Note: The close parenthesis comes directly after the range of x-values if there are no options.) Type in this command line (exactly as here) and press ENTER. (Until you enter either a semicolon or a colon and press ENTER, Maple will not do anything with your command. Also, if a command you want to enter doesn't all fit on one line, just keep typing, but don't press ENTER until you are finished. Maple automatically wraps around to a new line if you need it.)

If you make a typing mistake, Maple will let you know about it (!) One frequent source of errors in Maple sessions is unmatched parentheses, (or square or curly brackets, etc.) One "hacker's trick" is to count the open parens, count the close parens and make sure you get the same number (of course, if you don't, you also have to decide where to insert the missing one ...).

Fortunately, if this happens, the whole command does NOT need to be re-entered. Just move the cursor arrow to the place on the input line you want to change, press the left mouse button, and edit the input as needed. Typing from the keyboard will *insert* new stuff at the cursor location; the DELETE and BACKSPACE keys will remove stuff (DELETE removes the character in front of the "insert point"; BACKSPACE removes the character in back). You can also move around on the input line with the ARROW keys if more than one thing needs to be changed. When you think it's OK, press ENTER again to have Maple execute the command again. (Pressing ENTER with the cursor anywhere on an input line executes the whole input line.)

When all goes well you will see the graphics output displayed under the input command in the worksheet.

From the formula $y = x^4 - 2x^3 + x - 5\sin(x^2)$, you might guess that there is at least one other x-intercept for this graph for $x > 2$ (why?). To see that part of the graph as well, edit your previous command line to change the right hand endpoint of the interval of x values (do not retype the whole command). Press ENTER on that input line to have Maple execute the command again. Experiment until you are sure that your plot shows all the x-intercepts of this graph. (You can repeat this process of editing a command and re-running it as often as you want; the previous output is replaced by the new output each time.)

You can manipulate the graphics output *in place* within the worksheet in several ways. For instance:

1) If you click the left mouse button once over the graphics output, you will see a black box with eight "tabs" displayed at the corners and the midpoints of the edges of the box. If you place the cursor on one of the tabs, hold down the left mouse button, drag the cursor and release, you can *resize the graphics*. Try it!
2) Maple 2D graphics output regions have another nice feature: If you place the cursor arrow at a point in a 2D graphics region and click the LEFT mouse button once, *approximate coordinates* of the point at the head of the arrow are printed out in a box at the upper left of the window. How good approximations can you get that way to the x-intercepts of this graph?
3) Other features of the graph can be changed from the tool bar, which will have a new selection of buttons displayed after the graphics output is generated. Experiment and see if you can figure out what the different graphics tool bar icons do.

When you get a graphics window with *all* the intercepts shown, let's add one more thing. Sometimes, an informative title makes a graph much more understandable. To add a text title to a plot, you can insert a comma after the range of x-values, and include an option in the plot command of the form

```
title='whatever you want'
```

Add an appropriate title to your plot.

Example 2

Next, let's move to another 2D plot, to see the parametric curve $\alpha(t) = (t^2 - 1, t^3 - t)$. The basic format for parametric curves $\alpha(t) = (x(t), y(t))$ is

```
plot([x-comp,y-comp,t-range],options);
```

where

1) `x-comp` and `y-comp` are the x- and y-component functions of the curve to be plotted (each of these can be an *expression* involving the variable (parameter) t,
2) `t-range` is the range of t-values you want to see plotted, and
3) `options` can be used to control the form of the plot if desired. No options need be specified however if you don't want to.

Note that the x- and y-components and the t-range all go inside a set of square brackets $[,]$, separated by commas. This is also Maple's notation for a *vector, or ordered list*.

Using the `plot` command, as described above, display a plot of the curve $\alpha(t) = (t^2 - 1, t^3 - t)$ for $t \in [-3, 3]$.

Example 3

Frequently, it's the relationship between two or more different graphs that you want to understand by looking at a plot. You can also use the `plot` command to generate plots showing several graphs with the same coordinate axes. You do this by plotting an *unordered list* or set of functions. You do this by putting the formulas for the functions you want, separated by *commas*, between *curly braces* – {, } – where the single formula went before. (In fact this is the way sets or unordered lists are *always* entered in Maple.) For example, to plot $y = \sin(x)$ together with $y = \sin(x + 2)$ for $0 \leq x \leq 4\pi$, you could use:

```
plot({sin(x),sin(x+2)},x=0..4*Pi);
```

Try it. What is the general relationship between the graphs $y = f(x)$ and $y = f(x + k)$ for constant k? Explain.

Assignment – Due at end of Lab Period, Monday

Add a text region describing each of your plots from Examples 1,2,3 above and answering the questions asked above, then save and print this worksheet.

Days 2,3 – More Detailed Information on Maple Commands and Expressions, More Plotting

What is a Maple Command?

It may help to think of a command as a *program* or *function of one or more variables* that Maple knows how to compute, given "inputs" from you, although you must then keep straight the distinction between these functions Maple knows how to execute, and the functions or formulas you enter for Maple to work with!

Now that we have seen some first examples of Maple commands, here is some more information about the *syntax rules* that Maple uses to decide if what you have typed in is a well-formed command it can execute, and what the *semantics* or meaning of the commands are.

Almost all of the built-in Maple commands have a syntax like the `plot` command discussed above. The format is either

`name(values);`   or   `name(values):`

In this general description, `name` is the name of the command or function (like "plot" yesterday), there is a matching set of open and close parentheses following the name, and they surround the `values` – the input from you the command needs to do its job (like the formula of the function to be plotted and the range of x-values you want). The values are listed separated by commas within the parentheses. The two forms of the command above tell Maple to execute the command when you press ENTER on that input line. In the first case – ending the command with a semicolon (;) – you are instructing Maple to display the output. With colon (:), Maple will execute the command but not display the output (this is useful sometimes for intermediate steps in a big computation where you don't want or don't need to see the output).

Getting Help On-Line

The exact format Maple is expecting for each type of command is specified in the programming of the Maple system. Usually, there is *little or no freedom* in the ordering of what has to go where and in what format in the list of values. Much useful information on this for all the built-in commands, and LOTS of instructive examples are contained in the Maple on-line HELP facility. For example try looking at the on-line help page for the `plot command`. (From the HELP pull-down menu, select Topic Search, type in the word "plot," and press OK.) Note the example plot commands at the bottom and the links to related help pages.

Maple Expressions

Formulas or expressions are entered in something like usual mathematical notation:

1) The symbols for addition, subtraction, multiplication, and division are $+, -, *, /$ respectively.
2) The caret (^) is the Maple symbol for raising to a power.
3) The asterisk symbol for multiplication MUST be included whenever you are performing a product in a formula. Moreover, everything must be entered in one linear string of characters, so you will need to use *parentheses* to group terms to get the expressions you want. The rule to keep in mind is: *Maple always evaluates expressions by doing powers first, then products and quotients, then sums and differences, all left to right, unless parentheses are used to override these built-in rules.* For example, the Maple expression `a + b*c^2/d + e` is the same as the mathematical formula:

$$a + \frac{bc^2}{d} + e.$$

If you really want

$$\frac{a + bc^2}{d + e}$$

you will need to enter the expression `(a + b*c^2)/(d + e)`. What if you really wanted:

$$\frac{(a + bc)^2}{d + e}?$$

4) Maple "knows" all the usual elementary functions from calculus. The names of the most common ones are `sin, cos, tan, exp, ln, sqrt`. To use one of these functions in a Maple formula, you put the name, followed by the "argument" (that is, the constant or expression you are applying the function to) *in parentheses*.

Exercise 1 – Exercises 1 - 7 should be done in a single Maple worksheet, due no later than the end of the day on Friday

How would you enter each of the following formulas as Maple expressions? Try typing each in as a Maple input line, and compare with the output they generate:

1)

$$4xy^2 - 2x^3 + 3y - 7$$

2)

$$1 + \frac{1}{x^2} - \frac{x+2}{x^4}$$

3) (Note: `sqrt(x)` is the Maple syntax for $\sqrt{x}$.)

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

Implicit 2D Plotting

The 2D plots we generated yesterday were all either graphs of functions $y = f(x)$, or parametric curves. In addition to these, we will also want to be able to plot the affine variety

$$\mathbf{V}(g(x,y)) = \{(x,y) \in \mathbf{R}^2 : g(x,y) = 0\}$$

for a general polynomial $g(x,y)$ in two variables. The technical name for this in Maple is *implicit curve plotting*. To plot these curves in Maple, we need a new command called `implicitplot`. This command is not part of the basic Maple system; instead it is a part of a separate *package* of commands for various plotting operations called the `plots` package. Before you can use this command, you will need to "load" the plots package with the command

```
with(plots);
```

(The output is the list of commands in the package; you should see `implicitplot` about halfway along the list. If you don't want to see this each time, you can also put a colon after the `with` command as described above.)

The format for the `implicitplot` command is

```
implicitplot(expression,x-range,y-range);
```

The expression should be the equation $g(x,y)$ (Maple assumes you mean to plot $g(x,y) = 0$.) This command generates a plot of the part of the variety $\mathbf{V}(g(x,y))$ in the rectangular box in the plane defined by the x- and y-ranges.

Exercise 2

a) For instance, try the following command:

```
implicitplot(x^3 - 3*x*y^2 - 3,x=-3..3,y=-3..3);
```

to plot the variety $\mathbf{V}(x^3 - 3xy^2 - 3)$ Add a text region answering the following questions: How many intersections of this variety are there with vertical lines ($x = a$) and horizontal lines ($y = b$) in $\mathbf{R}^2$? Does the answer depend on the values of a, b? Is it possible to "see" these numbers from the form of the equation $g(x,y)$? Explain.

b) Same questions for $\mathbf{V}(x^3 - 3x + 2xy^2 - y^4 - 1)$

3D Curve Plotting

The `spacecurve` command in the `plots` package can be used to draw parametric curves in 3D. The format for the command is

```
spacecurve([x(t),y(t),z(t),t=a..b]);
```

where $x = x(t)$,$y = y(t)$,$z = z(t)$ are the parametric equations of the curve and the range of parameter values to be plotted is $a \le t \le b$. (Note the placement of the square brackets which follows the same pattern as the parametric version of `plot`!) For instance, try entering

```
spacecurve([t,t^2,t^3,t=-2..2]);
```

to plot the *twisted cubic* curve from class.

The first plot you see here might be rather uninformative. Fortunately, Maple also lets you add axes and look at a 3D plot from different viewpoints. Here's how:

1) Press the left mouse button over the graph. This should bring up a "bounding box" for the graph with corners you can *drag and drop* with the mouse to resize the graph.
2) If you place the mouse over the graph, depress the left button, and drag the cursor, you should see the graph rotating following the cursor position. Release the left mouse button when you get to the position you want.
3) If you click the right mouse button, you will bring up a menu containing the options `Style, Axes, Color` and several others. Select `Axes, Normal` with the left mouse button.

Practice a few times repositioning the viewpoint (rotating the viewing box) and redrawing the graph.

To draw several parametric curves in the same graph, you can include several curve specifications in the form `[x(t),y(t),z(t),t=a..b]` between curly braces `{,}`, separated by commas, all inside the parentheses for the `spacecurve` command.

Exercise 3

The line segment from (a, b, c) to (d, e, f) can be parametrized as follows

$$\alpha(t) = (a + t(d - a), b + t(e - b), c + t(f - c))$$

for $0 \le t \le 1$. The points $P = (0, 0, 0)$, $Q = (2, 0, 0)$, $R = (0, \sqrt{3}, 1)$, $S = (0, -1, \sqrt{3})$ are four corners of a cube in $\mathbf{R}^3$ with edges PQ, PR, and PS, because the vectors $\vec{u} = Q - P, \vec{v} = R - P, \vec{w} = S - P$ all have magnitude 2, and $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w} = \vec{v} \cdot \vec{w} = 0$. Your assignment, should you decide to accept it (just kidding!), is to create a picture of this cube by drawing the 12 edges together on the same set of axes in $\mathbf{R}^3$ (begin by drawing the line segments from P to Q, P to R, and P to S). Also explain how you found the other 4 corners of the cube. This can be done in a text region.

Surface Plots

a. Plotting a graph $z = f(x, y)$.

The command for 3D plotting is called `plot3d` (naturally enough!) Its format is similar to, but not exactly the same as the format of `plot`. To draw a graph with `plot3d`, you use a command of the format:

```
plot3d(function,xrange,yrange,options);
```

The `function` is the function $f(x, y)$, entered in the usual Maple syntax for expressions. The `xrange` and `yrange` specify a rectangular box in the plane that the plot will be constructed over; the `options` can be used to specify how the plot is drawn. Look at the online help for `plot3d[options]` if you want to see what things are possible. Most of the options can also be invoked from the 3D graphics output via the pop-up menu, or the toolbar buttons. Using the PATCHCONTOUR style is often a very effective way to visualize the shape of a surface.

For example, try

```
plot3d(-x^4+2*x^2-x/2-y^2-1,x=-2..2,y=-2..2);
```

to plot the surface

$$z = -x^4 + 2x^2 - x/2 - y^2 - 1.$$

One very nice feature of Maple 3D graphics is that you can change viewpoint, add axes, etc. using the mouse and the toolbar options in the 3D graphics window. The method is the same as that for the `spacecurve` described above.

Exercise 4

We will build up a "composite" Maple graphics display by combining a plot of a graph $z = f(x, y)$ produced with `plot3d` and a plot of a parametric curve produced using `spacecurve`. This can be done via the `display3d` command in the `plots` package. First we generate the plot of $z = x^2 - y^2$ for $(x, y) \in [-3, 3] \times [-3, 3]$, and assign the output to the variable `P1`.

```
P1:=plot3d(x^2-y^2,x=-3..3,y=-3..3):
```

(The colon at the end of the input line *suppresses the output*, which is Maple's internal form of the graph. To see why I did that, try replacing the colon with a semicolon and pressing RETURN again on that line to reexecute!) Then, we generate a plot of the parametric curve $\alpha(t) = (t, t^2, t^3)$ for $t \in [-2, 2]$, drawn with a relatively thick line and in black for contrast, and put the results into the variable `P2`:

```
P2:=spacecurve([t,t^2,t^3,t=-2..2],color=black,thickness=2):
```

Finally, we display both graphs together:

```
display3d({P1,P2});
```

(Here of course, you *do* want to see the output, presumably.) Maple is smart enough about these plots to combine the scales correctly and show the curve and surface in their proper relative positions! Look at this one for a while until you see how the curve and the surface are intersecting. How many intersection points are there?

Any number of different 3D plots can be displayed together in this way by assigning each to a variable, then putting the set or list of plot variables names in the `display3d` command.

b. Implicit Surface Plots

To plot an surface defined as a variety $\mathbf{V}(g(x, y, z))$, you can use the `implicitplot3d` command. Look up the on-line help listing for this command and use it in the following exercise:

Exercise 5

Generate a plot of the following variety in $\mathbf{R}^3$:

$$S = \mathbf{V}(z^2 - (16 - x^2 - y^2)((x + 2)^2 + y^2 - 1)((x - 2)^2 + y^2 - 1)/10)$$

and explain the shape you see. You will want to "walk around" this one a lot by rotating and looking at it from different viewpoints (For example, which (x, y) correspond to points on S and why? You will also need to think about how you choose the x-, y-, and z-ranges. Be sure you take the x- and y- ranges big enough to see all the points on the variety.) Suggestion: Use the options `style=PATCHCONTOUR` and `grid=[30,30,30]` to see detail. Be aware though that this larger than normal number of grid points will mean the surface takes longer to compute and "render" or draw and your worksheet will require more space to store on disk if you save the graphics.

Day 4: Symbolic Computation in Maple

Much of our work starting next week in Maple will consist of *symbolic* computations in Maple – mainly various manipulations of polynomials in several variables. Today, we will look at some first examples of this sort of computation in Maple, with polynomials in one variable.

The assignment operator :=

We have already run into this briefly when we combined two plots in the last lab. In symbolic computation, we will use this idea all the time. A Maple command of the form

```
name := expression;
```

assigns the right hand side to the name on the left. Usually, the right hand side will involve a known expression, or performing some operation on some information we already know. We want to take the result of the operation and give it an abbreviated "name" for later use. For instance,

```
cube := (x^2 - 16)^3;
```

Then later on, we could "reuse" the value of `cube` in other expressions, just by putting in the name `cube` at the appropriate place (e.g. `plot(cube, x=5..10);`). Note: Maple also gives you a second way to refer back to results of previous computations. At any point in a session, if you enter %, this stands for the result of the previously executed command, %% stands for the result of the command before that, and so on. (Caution: The sequence is the sequence of the commands you have executed in the current session, *not* the sequence of input lines in the worksheet. This can be confusing if you skip around, inserting new command lines between others.)

Some additional symbolic commands on polynomials

Maple has built-in commands:

- `expand` to multiply out (and also simplify) a factored polynomial. Format: `expand(poly);` This works on polynomials in any number of variables.
- `factor` to factor a polynomial. This gives the factorization with *rational number coefficients*. The factor command does not know about radicals, complex numbers, etc. Format: `factor(poly);` This works on polynomials in any number of variables.
- `quo` to compute the quotient on division of one polynomial $f(x)$ by another $g(x)$. Format: `quo(f,g,x);` Note: the variable must be included. If f or g contain other variables besides x, they will be treated as constants in the coefficients.
- `rem` to compute the quotient on division of one polynomial $f(x)$ by another $g(x)$. Format: `rem(f,g,x);` Note: the variable must be included again.
- `gcd` to compute the greatest common divisor of two polynomials $f(x)$ and $g(x)$. Format: `gcd(f,g);` Note: the variable is *not* included in this one. The greatest common divisor of a set of several polynomials can be computed by nested gcd's (see p. 43 of "IVA"; for example $gcd(f, g, h)$ can be computed by `gcd(f,gcd(g,h))`.
- `diff` to compute derivative of a polynomial (or any other function) with respect to a given variable. Format `diff(f,x);`

Here is a sample sequence of Maple input lines illustrating some of these commands. Try entering and executing them in sequence. Look carefully at the output and make sure you understand what happened in each case:

```
cube := (x^2 - 16)^3;
diff(cube,x);
cubee:=expand(cube);
factor(cubee);
d:= x^2-x+1;
q:=quo(cubee,d,x);
r:=rem(cubee,d,x);
factor(expand(q*d+r));
```

Exercise 6

Using Maple and the commands above, do problems 9,12,14,15 from Chapter 1, §5 of "IVA".

A First Taste of Maple Programming

In addition to a large collection of built-in commands for performing various mathematical computations, Maple also contains a *programming language* that you can use to code and implement your own new commands or procedures. Here is a first example of a Maple procedure which takes as input two polynomials in one variable, $f(x)$ and $g(x)$, and computes their gcd $h(x)$, together with polynomials $A(x)$ and $B(x)$ satisfying

$$h(x) = A(x)f(x) + B(x)g(x)$$

(recall this was question 10 from Chapter 1, §5 of "IVA"). Here is Maple code for one way to do this:

```
ExtEuc := proc(f,g,x)
    local A,B,C,Csave,D,Dsave,q,r,s,h;
    h := f; s:= g; A := 1; B := 0; C := 0; D := 1;
    while s <> 0 do
        q := quo(h,s,x);
        r := rem(h,s,x);
        h := s;
        s := r;
        Csave := C; Dsave := D;
        C := expand(A - q*C); D := expand(B - q*D);
        A := Csave; B:= Dsave;
    od;
    RETURN(h,A,B)
    end:
```

Note that the syntax here is very similar to our pseudocode for algorithms. There is a file containing the above Maple code in our class directory. You can read it into your Maple session using:

```
read 'f:/users/simu/Little/ExtEuc.map';
```

Then to use the procedure to compute the gcd and the polynomials A, B, you would enter a command like this:

```
ExtEuc(fpoly,gpoly,var);
```

where `fpoly` and `gpoly` are the two polynomials, and `var` is the variable. The output from the procedure consists of the last value of h (the gcd, up to a constant multiple), then the polynomials A and B. Use this to check your work from this morning.