Mathematics 242 – Principles of Analysis
Midterm Exam 3
May 2, 2014

Directions

Do all work in the blue exam booklet. There are 100 possible regular points and 10 possible Extra Credit points (see question IV below). Possibly useful information:

$$\sum_{i=1}^{n} = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

I. (15) Let

$$f(x) = \begin{cases} x^{4/5}\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Is f continuous at $x = 0$? Is f differentiable at $x = 0$? Give complete reasons for your assertions.

II. Both parts of this question refer to the function $f : \mathbf{R} \to \mathbf{R}$ defined by $f(x) = 1 - x^2$.

A) (20) Consider the regular partitions $\mathcal{P}_n$ of the interval $[1, 3]$ and show directly, using the upper and lower sums, that f is integrable on $[1, 3]$.
B) (15) Explain why the hypothesis of the Mean Value Theorem is satisfied for f on the interval $[1, 3]$ and find the number c mentioned in the conclusion.

III. (20) Prove that if f is continuous on $[a, b]$, then f is integrable on $[a, b]$.

IV. True-False. Say whether each of the following statements is true or false. For true statements, give short proofs; for false ones give reasons or counterexamples. Do any *three* parts. If you submit solutions for all four, then I will consider the other one for Extra Credit.

A) (10) Let $f(x) = e^{2x} - e^x$. There exists some $c \in (0, \ln(2))$ such that $f(c) = 1$.
B) (10) The function $f(x) = \arctan(x)$ is uniformly continuous on the interval $(-1, 1)$.
C) (10) There are continuous functions $f(x)$ on $[a, b]$ for which there exist no differentiable function $F(x)$ on $[a, b]$ with $F'(x) = f(x)$.
D) (10) Let f be differentiable on an open interval I with $[a, b] \subset I$. If $f'(a) > 0$ and $f'(b) < 0$, then there must exist some $c \in (a, b)$ where $f'(c) = 0$.