Mathematics 242 – Principles of Analysis
Makeup Exam 2 – April 4, 2014

Directions

Do all work in the blue exam booklet. There are 100 possible regular points and 10 Extra Credit Points.

I.

A) (20) State and prove the Monotone Convergence Theorem for sequences. (You may give the proof in the case that the sequence is monotone increasing.)

B) (10) Let $\{x_n\}$ be the sequence defined by $x_1 = 5$ and

$$x_{n+1} = \frac{12 + x_n^2}{15}$$

for all $n \geq 1$. Does this sequence converge? Why? If it does, what is the limit?

II.

A) (15) Show using the ε, n_0 definition that

$$\lim_{n\to\infty} \frac{4n+1}{7n+3} = \frac{4}{7}.$$

B) (15) Show using the ε, δ definition that

$$f(x) = \frac{4x+1}{7x+3}$$

is continuous at $x = 0$.

III. (10) Suppose $\{x_n\}$ is a sequence such that $|x_n - 10| < 30$ for all $n \geq 1$. Show that there exists some number $a \in [-20, 40]$ and a subsequence $\{x_{n_k}\}$ such that $x_{n_k} \to a$. State any "big theorems" you are using.

IV. Give an example, or give a reason why there can be no such examples:

A) (10) A function f and an a in the domain of f such that $\lim_{x\to a^-} f(x)$ does not exist, but $\lim_{x\to a^+} f(x)$ does exist.

B) (10) A sequence x_n such that $x_n \to -4$, but for all $n_0 \in \mathbf{N}$ there exist $n \geq n_0$ with $x_n > 0$.

C) (10) A bounded function that is not continuous at any c in its domain.

Extra Credit. (10) Assume that $\{x_n\}$ is a sequence that converges to a. Construct a new sequence $\{y_n\}$ by making y_n the average of the terms $x_n, x_{n+1}, x_{n+2}, x_{n+3}, x_{n+4}$ in the original sequence. So for instance,

$$y_1 = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}, y_2 = \frac{x_2 + x_3 + x_4 + x_5 + x_6}{5},$$

and so on. Show that $\{y_n\}$ also converges to a.