Mathematics 242 – Principles of Analysis
Information on Final Examination
December 1, 2000

General Information

- The final examination for this class will be given during the scheduled period – 2:30 to 5:30 pm on Friday, December 15.
- The final will be a *comprehensive exam*, covering all the topics from the two midterms, *and the material about infinite series from the past two weeks.* See the list of topics below for more details.
- The exam will be similar in format to the midterms but roughly twice as long – it will written to take about 2 hours if you work steadily, but you will have the full 3 hour period to use if you need that much time.
- If there is interest, I would be happy to arrange an evening review session during exam week (Wednesday would probably be the best day). We can discuss this in class on December 1.

Philosophical Comments and Suggestions on How to Prepare

- The reason we give final exams (either in-class or take-home) in almost all mathematics classes is to encourage students to "put whole courses together" in their minds. Also, preparing for the final should make the ideas "stick" so you will have the material at your disposal to use in later courses.
- If you approach preparing for a final exam in the right way it can be a *real learning experience* – especially in a class like this one where almost everything we have done "fits together" in a very tight chain of logical reasoning starting with the LUB axiom for the real number system. Much of what we did earlier in the semester should make much more sense now than it may have at the time!
- But this time there is a period of almost 2 weeks between the end of classes and our exam. As if that weren't bad enough, as you all probably know, it is very easy to get "burned out" toward the end of exam week. The fact that our exam is so late is unfortunate, but it's a situation you can handle with proper planning. *START REVIEWING NOW* and do some review each day between now and December 15 (even just 1/2 hour each day will make a big difference). That way you will not be "crunched" at the end (and with any luck the ideas we have developed in this course will "stick" better!)

Topics To Be Included

1) The real number system, rational and irrational numbers, the algebraic and order properties, the least upper bound axiom.
2) Mathematical induction
3) Sequences, convergence ($\lim_{n\to\infty} x_n$ – both ϵ - n_0 definition, and computing limits via the "Big Limit Theorem".

4) Subsequences, The Nested Interval Theorem, and the Bolzano-Weierstrass Theorem
5) Limits of functions, the limit theorems (sum, product, quotient, "squeeze" rules, one-sided limits
6) Continuity, the Extreme and Intermediate Value Theorems
7) Definition and properties of the derivative, the Mean Value Theorem and its consequences
8) The definite integral, integrability, the Fundamental Theorem
9) Infinite series – convergence and divergence, key examples such as geometric series, p-series, etc. Absolute vs. conditional convergence. Comparison, integral, alternating series, and ratio tests for convergence.

Definitions and Theorems to Know

Be able to give careful statements of the following definitions/axioms and theorems:

1) The Least Upper Bound Axiom
2) The definition of convergence for a sequence
3) The definition of convergence for a series
4) The definitions of $\lim_{x \to c} f(x) = L$ and f is continuous at c
5) The Extreme and Intermediate Value Theorems
6) The Ratio Test for absolute convergence of a series

Also be able to give proofs of the following:

1) Every monotone increasing sequence of real numbers that is bounded above converges.
2) Prove that a given sequence x_n converges to a limit a, using the ϵ - n_0 definition
3) Statements of the form $\lim_{x \to c} f(x) = L$ using the ϵ - δ definition
4) The Mean Value Theorem
5) The Fundamental Theorem of Calculus

Suggested Review Problems

See review sheets for Midterm Exams 1 and 2 for topics 1-8 in the list above. For topic 9:

Section 6.1/1, 3, 4, 8, 9
Section 6.2/1, 4, 5