Plato's Criticism of Geometers of His Time
From Plutarch, *Quaestiones Convivales*, Book 8, Chapter 2, Section 1
Text for Oral Presentation

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The Plutarch passage I will discuss comes from a section of the *Moralia* known as the Συμποσιακά, or *Quaestiones Convivales*, or "Table Talk." Each section of this work is presented as a record of conversation at συμπόσια or drinking parties arranged by Plutarch for various of his guests. It is amusing to see what he says about the rationale for the sort of questions considered: in "... our entertainments we should use learned and philosophical discourse ... such discourse being applied to drunkenness, every thing that is brutish and outrageous in it [i.e. drunkenness!] is concealed ... ." To keep your next party from degenerating into a drunken brawl, try this!

In Book 8, Chapter 2, section 1, during a celebration of Plato's birthday, Plutarch presents a conversation concerning the role of the study of geometry in Plato's thought. The guest Diogenianus begins by raising the question why Plato asserted that "God always geometrizes." He says he is not aware of any specific text where Plato said precisely that, though it sounds like something Plato would have said. Another guest, Tyndares, replies that there is no great mystery becuase Plato had often written that geometry "takes us away from the sensible and turns us back to the eternal nature we can perceive with our minds, whose contemplation is the goal of philosophy... " and presents an interesting piece of evidence:

"δίο καὶ Πλάτων αὐτὸς ἐμέμψατο τοὺς περὶ Εὔδοξον καὶ Ἀρχύταν καὶ Μέναιχμον εἲς ὀργανικὰς καὶ μηχανικὰς κατασκευὰς τὸν τοῦ στερεοῦ διπλασιασμὸν ἀπαγεῖν ἐπιχειροῦντας ... ." "Therefore even Plato himself harshly criticized Eudoxus, Archytas, and Menaechmus for attempting to reduce the *duplication of the cube* to mechanical constructions with instruments ... . "

The *duplication of the cube* was a geometrical problem asking for the construction of the side of a cube whose volume would be twice the volume of a given cube–one of a series of geometric construction problems that stimulated much of the development of Greek mathematics through the Classical period. Eudoxus of Cnidus (409–356 BCE), Archytas of Tarentum (428–347 BCE), and Menaechmus of Alopeconnesus (380–320 BCE) were three of the most accomplished Greek mathematicians active in the 4th century BCE. Archytas is often identified as a Pythagorean and there are traditions that Eudoxus was a pupil of his and Menaechmus was a pupil of Eudoxus, all three being associated with Plato and his Academy in Athens in some way.

Tyndares continues in a surprisingly technical vein, "ὥσπερ πειρωμένους δι᾽ ἀλόγου δύο μέσας ἀνάλογον, ᾗ παρείκοι, λαβεῖν ... ".
I propose this reading: "just as though they were trying, in an unreasoning way, to take two mean proportionals in continued proportion any way that they might ... ". The δι᾽ ἀλόγου is hard to translate and may not even be what Plutarch originally wrote since this specific phrase has a rather large number of textual issues. It is interesting that Tyndares seems to be assuming that all of his listeners will be familiar with this terminology.

The "two mean proportionals" refers to a key piece of progress on the *duplication of the cube* made somewhat before the time of Plato by Hippocrates of Chios (ca. 470–ca. 410 BCE). Given two line segments $AB$ and $GH$, we say line segments $CD$ and $EF$ are *two mean proportionals in continued proportion* between $AB$ and $GH$ if their lengths satisfy:

$$\frac{AB}{CD} = \frac{CD}{EF} = \frac{EF}{GH}. \tag{1}$$

In the Plutarch passage, this appears in the accusative as δύο μέσας ἀνάλογον. The ἀνάλογον seems to be essentially equivalent to ἀνὰ λόγον – "in (continued) proportion."

Hippocrates' contribution was the realization that if

$$GH = 2AB,$$

then some simple algebra shows

$$CD^3 = 2AB^3.$$

In other words, if $AB$ is the side of the original cube, then $CD$ is the side of the cube with twice the volume. This was not a full solution for the *duplication of the cube*, but it did provide a way to attack the problem and almost all later work took this as its starting point.

Tyndares concludes his summary of Plato's criticism by claiming that these mechanical procedures with tools would have the effect "ἀπόλλυσθαι γὰρ ... καί διαφείρεσθαι τὸ γεωμετρίας ἀγαθὸν αὖθις ἐπὶ τὰ αἰσθητὰ παλινδρομούσης καὶ μὴ φερομένης ἄνω μηδ′ ἀντιλαμβανομένης τῶν ἀίδιων εἰκόνων αἷσπερ ὢν ὁ θεὸς ἀεὶ θεός ἐστι."
"... to destroy utterly the good of geometry and again turn it around to things of the senses, not above to the eternal forms, being in which, God is always God."

Tyndares is saying that Plato criticized the mechanical nature of the solutions proposed by Eudoxus, Archytas, and Menaechmus because they in effect subverted what Plato saw as the true purpose of geometry, which was not merely to solve problems "by any means necessary," but rather to lead the soul to the contemplation of eternal truth.

How might the adjectives μηχανικός or ὀργανικός apply to geometric constructions? On the face of it, ὀργανικός, in the sense of instrument-based, or tool-based, is clearer. For the adjective ὀργανικός to apply, I believe some physical device must be involved in the construction. But even there, there is a slightly subtle point. The Greeks, even though they almost certainly used physical straightedges to draw lines and physical compasses to draw circles while constructing diagrams, were apparently also happy to consider those tools in *idealized versions* (e.g. in the first three Postulates in Book I of Euclid's *Elements*). What "counts" as μηχανικός is unfortuately even less clear. One possibility is a construction that has some element of *physical or imagined motion*. Note that change over time in a figure would itself violate Plato's vision of the eternal and unchanging nature of the world of the forms.

Plutarch does not include any discussion of what Eudoxus, Archytas, or Menaechmus actually did to to find the two mean proportionals between two given line segments. However, accounts of the work on this problem including information about their approaches have survived in an ancient source: a much later commentary on Archimedes' *On the Sphere and Cylinder* by Eutocius of Ascalon (ca. 480 – ca. 540 CE). It's interesting (but also very subtle) to try to see to what extent the adjectives μηχνικός or ὀργανικός applies to them, and to what extent Plutarch's account of Plato's criticism might refer to actual history. If anyone is interested in the details, I would be happy to share the fuller write-up I have prepared for my final paper.

In conclusion, we can say that Plutarch has seemingly preserved a largely accurate picture of Plato's thinking. But from the work of Archytas and Menaechmus and the later work of Archimedes, Apollonius and others it is also clear that if something like Plato's criticism of the geometers in his circle actually happened, its effect on the rapidly-developing state of knowledge in Greek mathematics at this point was somewhat minimal. By the time this episode supposedly took place mathematics had emerged as an independent subject and Plato's ideas about what its proper methods or goals were were not the last word.