Mathematics 304 – Ordinary Differential Equations
Problem Set 1
September 7, 2000

Information: This is an individual assignment, due in class Thursday, September 14.

From the text:

Section 2.2 (p. 51-53)/12, 17, 19, 21, 29, 31

Section 2.3 (p. 59-60)/1, 3, 5, 15, 17

Extra Problems

I) In question C on Lab 1, we graphed some solutions of the ODE with piecewise-defined slope function

$$\frac{dP}{dt} = \begin{cases} (0.4)P(1 - P/30) & \text{if } t \le 5 \\ (0.4)P(1 - P/30) - (0.25)P & \text{if } t > 5 \end{cases} \tag{1}$$

Note that the Existence and Uniqueness Theorem does not guarantee anything about solutions on intervals containing $t = 5$, since the slope function is *discontinuous* at (t, P) with $P > 0$. Nevertheless, Maple did not seem to have any difficulty generating plots of approximate solutions for this equation. We want to understand this, and generate formulas for the solutions Maple found.

A) Explain, using the Existence and Uniqueness Theorem, how a *continuous* solution of the equation (1) could be "pieced together" from a solution to $\frac{dP}{dt} = (0.4)P(1-P/30)$, with an initial condition at $t = 0$, and a solution to $\frac{dP}{dt} = (0.4)P(1-P/30) - (0.25)P$, with an initial condition at $t = 5$.

B) Both equations in A are separable *logistic equations* as we discussed in class on 9/7. Find explicit solutions of each and piece together the appropriate solution of (1) with $P(0) = 25$. This is one of the cases you did in Maple in the lab. How does the plot of your explicit analytic solution compare with the approximate plot Maple found?

C) In two or three sentences, describe the effect of the appearance of the parasite on the insect population at $t = 5$. Explain the behavior as $t \to \infty$.

II) We can also get information on the second derivative (concavity) of solutions of a first order ODE $\frac{dy}{dx} = f(x, y)$ by differentiating both sides with respect to x and using *implicit differentiation* on the right-hand side – that is differentiate $f(x, y)$ with respect to x, thinking of y as the solution function, so y depends on x.

A) Using this, show that the graph of the solution of $\frac{dy}{dx} = xy$ with $y(0) = 1$ is concave up for all x.

B) Show that every solution of the autonomous equation $\frac{dy}{dx} = (y-2)(y-4)$ and $y(0) = y_0$ with $2 < y_0 < 4$ has an inflection point where it crosses the line $y = 3$.