Mathematics 304 – Ordinary Differential Equations
Review Sheet – Final Examination
November 30, 2000

General Information

- The final examination for this class will be given during the scheduled period – 2:30 to 5:30 pm on Thursday, December 14.
- The final will be a *comprehensive exam*, covering all the topics from the two midterms, *and the material about linearization of 1st order autonomous systems, solutions of 1st order linear systems with constant coefficients, and classification of critical points from the last 3 weeks.* See the list of topics below for more details.
- The exam will be written to take about 2 hours if you work steadily, but you will have the full 3 hour period to use if you need that much time.
- Like the second in-class midterm, this will be an *open book exam.* This means you do not have to memorize everything, *but it also means that I can ask a "curve ball" question where you have to think a bit (take what you know and apply it in a new situation).*
- If there is interest, I would be happy to arrange an evening review session during exam week (Monday or Tuesday would probably be the best days). We can discuss this in class on November 30.

Topics To Be Included

The exam will cover the following topics from Chapters 1-5, 9, 12 of the text:

1) Terminology of ODE, solutions, initial value problems, the Existence and Uniqueness Theorem for solutions of first order initial value problems (one equation, one dependent variable).
2) Qualitative theory for 1st order ODE: Direction Fields, special techniques for autonomous equations (identifying equilibrium solutions, determining whether they are "stable" or "unstable" – see section 1.4 in the text and Discussion 1)
3) Techniques for deriving analytic solutions of 1st order ODE: Separable equations, linear equations, exact equations, integrating factors, substitutions and transformations for homogeneous equations, equations of the form $\frac{dy}{dx} = g(ax + by + c)$, Bernoulli equations.
4) 1st order ODE in modeling of mixing problems, heating/cooling, etc.
5) The theory of second order linear ODE: the existence and uniqueness theorem for solutions, fundamental sets of solutions for homogeneous equations, the Wronskian and Abel's identity, general solutions of inhomogeneous equations.
6) The constant coefficient case – the auxiliary equation, forms for solutions when the roots are real or complex.
7) Superposition and the method of undetermined coefficients for inhomogeneous equations.
8) Applications to mass-spring systems, electrical circuits, etc. Resonance phenomena.
9) First order systems – critical points, the phase plane, trajectories and integral curves.

10) First order systems with constant coefficients – solutions via eigenvalues, eigenvectors of the coefficient matrix.
11) Linearization of a first order system at a critical point, classification of critical points in terms of eigenvalues of the linearized system (see sections 12.2, 12.3 in text)

Suggested Review Problems

See review sheets for Midterm Exams 1 and 2 for topics 1-9 in the list above. For topics 10, 11:

Section 9.5/17, 18, 19, 21, 25, 35, 37, 42, 43
Section 9.6/1, 3
Section 12.2/1, 7
Section 12.3/9, 11