Mathematics 41, section 1 – Multivariable Calculus
Information on Exam 3
November 13, 1999

General Information

The third exam for the course will be given Friday, November 19, as announced in the course syllabus. It will cover the material discussed in class since immediately before the second exam through class on Monday, November 15.

Topics to Be Covered

a) Critical points for functions $f(x, y)$, the "First Derivative Test" for local maxima and minima – see the Proposition on page 293 of the text (section 4.1)
b) The Second Derivative Test for local maxima and minima of functions (section 4.2)
c) The Lagrange Multiplier method for constrained max-min problems (section 4.3)
d) Riemann sums and double integrals over rectangular regions in $\mathbf{R}^2$ (section 5.1)
e) Double Integrals over non-rectangular regions via iterated integrals with variable limits of integration (section 5.2)

There will again be 3 or 4 problems, possibly with several parts. Be prepared to:

1) State the First and Second Derivative Tests (see class notes, and/or the Propositions on page 293, 318 of the text), and use them to classify critical points of a function.
2) Set up and solve the Lagrange Multiplier equations for constrained max-min problems
3) Use a Riemann sum to get an approximate value for a double integral, and
4) Use exact integration methods like substitution, integration by parts, etc. to evaluate double integrals

Review Session

I hope to be able to schedule a problem session before the exam, but I have some heavy schedule constraints this time. Wednesday and Thursday in the early evening are not possible for me. I have the Chamber Orchestra concert on Wednesday and another rehearsal in Boston on Thursday evening. Wednesday late afternoon (say 4 - 6 pm) might work. Wednesday at 9:30 pm or shortly thereafter would also be possible. Thursday 3:30 - 5:30 pm is also possible, but that's getting pretty close to the exam!

Suggested Practice Problems

From the text:
Section 4.1/4,5,6,11,12
Section 4.2/3,7,10,11
Section 4.3/3,4,5,6
Section 5.1/3,4,5
Section 5.2/1,2,4,6,7,12,13,14

Practice Exam

I. All parts of this problem refer to $f(x, y) = x^3 + y^2 - 3x$.

A) Find all critical points, and classify them using a plot of the gradient vector field.
B) Use the Second Derivative test to verify your work from part A.
C) Use the Lagrange Multiplier method to find the maxima and minima of f on the constraint curve $x^2 + y^2 = 9$.
D) What are the biggest and smallest values of f on the region $\{(x, y) : x^2 + y^2 \leq 9\}$?

II.

A) At the corners, the midpoints of the sides, and the center of the rectangle $R = [1, 3] \times [0, 2]$ in the xy-plane, the values of a function $f(x, y)$ are measured to be

$$\begin{array}{lll} f(1,2) = 6 & f(2,2) = 8 & f(3,2) = 10 \\ f(1,1) = 3 & f(2,1) = 5 & f(3,1) = 7 \\ f(1,0) = 2 & f(2,0) = 4 & f(3,0) = 6 \end{array}$$

Estimate $\int\int_R f(x, y)\, dA$ using an appropriate Riemann sum.

B) The function $f(x, y)$ from part A was $f(x, y) = 2x + y^2$. Compute $\int\int_R f(x, y)\, dA$ exactly.

III. A region S in the first and second quadrants of the xy-plane is bounded by the lines $y = x, y = -x$, and by an arc of the circle $x^2 + y^2 = 9$.

A) Find the limits of integration to set up $\int\int_S y\, dA$ as one or more iterated integrals of the form $\int\int y\, dydx$. DO NOT evaluate yet.
B) Now find the limits of integration to set up $\int\int_S y\, dA$ as one or more integrals of the form $\int\int y\, dxdy$. DO NOT evaluate this integral yet either.
C) Evaluate the integral of your choice from either A or B.