Mathematics 241, section 1 – Multivariable Calculus
Exam III – November 26, 2013

Directions

Do all work in the blue exam booklet. There are 100 regular points and 10 Extra Credit points.

I.
A) (10) Give a precise statement of the Second Derivative Test for local extrema of functions of two variables.
B) (20) Let $f(x, y) = \frac{x}{x^2+y^2+1}$. Find all critical points of f and apply the Second Derivative Test to each.

II.
A) (20) Find the maximum and minimum values of

$$f(x, y) = xy + 3y$$

on the closed and bounded set

$$S = \{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 9\}.$$

B) (5) Give a sketch of the contours of f for the maximum and minimum values you found in part A, together with S.

III. All parts of this problem refer to the region

$$R = \{(x, y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 4, y \leq 0, y \geq 2x\}.$$

and the solid $S \subset \mathbf{R}^3$ with "shadow" R in the xy-plane and $0 \leq z \leq x^2$.

A) (5) Sketch the region R in the plane.
B) (10) Set up iterated integral(s) of the form $\int\int f(x, y)\ dy\ dx$ to compute the volume of S. Do not compute the value.
C) (10) Now set up iterated integral(s) of the form $\int\int f(x, y)\ dx\ dy$ to compute the volume of S. Do not compute the value.
D) (10) Finally, set up a polar coordinate integral that will compute the volume of S. Do not compute the value.
E) (10) Finally, compute the volume of S using any one of your answers from parts B,C,D (your choice). *For full credit, you must show all work leading to your final answer.*

Extra Credit (10) Evaluate the improper double integral

$$\int\int_{\mathbf{R}^2} e^{-x^2-y^2}\ dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2-y^2}\ dy\ dx.$$