Mathematics 241, section 1 – Multivariable Calculus
Makeup Midterm Exam 2
November 1, 2013

Directions

Do all work in the blue exam booklet. There are 100 regular points and 10 Extra Credit points.

I. All parts of this problem refer to the vector field

$$\mathbf{F}(x, y) = (x^2 - 2x, xy - y).$$

A. (10) Find all critical points of $\mathbf{F}(x, y)$.

B. (5) There are two vector fields plotted on the back of this sheet. Say which one shows $\mathbf{F}(x, y)$ and use that plot to classify each of the critical points as a source, sink, saddle, or center.

C. (20) Show that $\alpha(t) = (0, 4e^{-t})$ and $\beta(t) = \left(\dfrac{2}{1 + e^{2t}}, 0\right)$ are both flow lines of F. What are $\lim_{t\to\infty} \alpha(t)$ and $\lim_{t\to\infty} \beta(t)$?

D. (5) Is there a scalar-valued function $f(x, y)$ such that $\mathbf{F}(x, y) = \nabla f(x, y)$? Why or why not?

II. In the neighborhood of Eagle Mountain, the landscape has elevation in feet above sea level given by $f(x, y) = \frac{x^2}{4} + y^2 + 1000$.

A. (10) Sketch the contours of $f(x, y)$ for $c = 999, 1000, 1001$ on the same set of axes.

B. (10) Compute the directional derivative $D_u f(2, 1)$ for a general unit vector.

C. (5) In the direction of which unit vector u should you walk from the point with $(x, y) = (2, 1)$ in order to decrease your elevation at the fastest rate?

III. All parts of this problem refer to the function

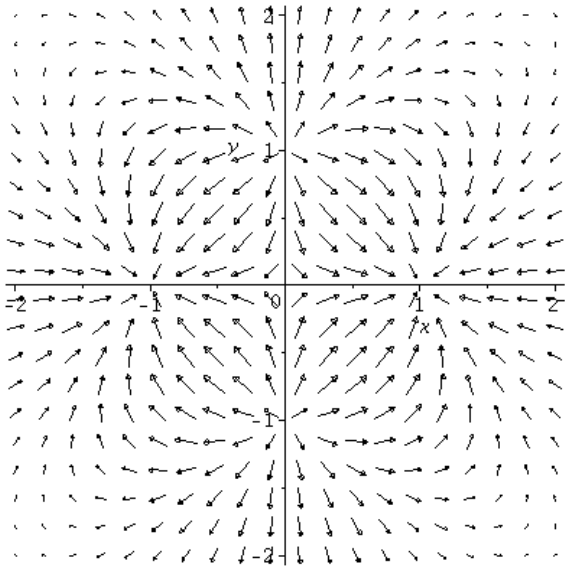
$$f(x, y) = \frac{x^3 + x^2 y}{x^2 + y^2} \text{ if } (x, y) \neq (0, 0) \text{ and } f(0, 0) = 0.$$

A. (15) Find the tangent plane to $z = f(x, y)$ at $(1, 1, f(1, 1))$.

B. (10) Does $\frac{\partial f}{\partial x}(0, 0)$ exist? If so, find it; if not say why not.

Extra Credit (10) Refer to the function in question III. Let m be arbitrary and compute $\lim_{t\to 0} f(t, mt)$ (the limit of the value of f along the line through the origin in the direction of the vector $(1, m)$). Is $\lim_{t\to 0}(f \circ \alpha)(t) = 0$ for *every* differentiable curve $\alpha(t)$ with $\alpha(0) = (0, 0)$? Explain.

Vector Field 1:



Vector Field 2: