Mathematics 241, section 1 – Multivariable Calculus
Information on Exam 3
November 18, 2013

General Information

The third exam for the course will be given Tuesday, November 26 (the day before Thanksgiving break), as announced in the course syllabus. It will cover the material from Problem Sets 7, 8, and 9. As always, though, *there are major dependencies of this material on things we discussed earlier in the semester. You should, in fact, be prepared to do any sort of computation we have covered, including vector operations, finding equations of planes, finding parametrizations, determining critical points of vector fields, etc.)*

Topics to Be Covered

a) The second degree Taylor polynomial and the Second Derivative Test for critical points of functions (section 4.2)
b) The Lagrange Multiplier method for constrained max-min problems (sections 4.3 and 4.4), including max-min questions on closed, bounded subsets of the plane or space.
c) Riemann sums and double integrals over regions in $\mathbf{R}^2$ (section 5.1)
d) Double Integrals over regions via iterated integrals with variable limits of integration – Fubini's Theorem (section 5.2) with applications to volumes, average values, centers of mass.
e) Polar coordinates in the plane and polar double integrals, converting rectangular double integrals to polar coordinates – the Change of Variables Theorem (section 5.3)
f) Triple integrals (section 5.4). Note that finding a volume using polar coordinates in the xy-plane and the rectangular z-coordinate is the same as using cylindrical coordinates. So there is a big overlap between things from sections 5.3 and 5.5 and a lot of questions in section 5.5 are things I could ask on this exam too.

There will again be 3 or 4 problems, possibly with several parts. Be prepared to:

1) State the Second Derivative Test (see class notes, and/or the Proposition 4.6 on pages 242-243 of the text), and use it to classify critical points of a function.
2) Set up and solve the Lagrange Multiplier equations for constrained max-min problems,
3) Use the Second Derivative Test and some Lagrange Multiplier calculations to find the maximum and/or minimum values of a function on a closed set.
4) Use a Riemann sum to get an approximate value for a double integral, and
5) Use exact integration methods like substitution, integration by parts, etc. to evaluate double integrals.

Review Session

We can review for the exam in class on Monday, November 25.

Suggested Practice Problems

From the text:
End of Chapter 4 Exercises/1,2,3,8,12b
End of Chapter 5 Exercises/1, 5, 6ab, 7abcde

Practice Exam Questions

Note: As usual, the actual exam questions may be organized differently and may include different types of examples. The actual exam will also be somewhat shorter.

I. All parts of this problem refer to $f(x, y) = x^3 + y^2 - 3x$.

A) Find all critical points of f.
B) Give a precise statement of the Second Derivative Test.
C) Use the Second Derivative test to classify each critical point of f as a local maximum, local minimum, or a saddle point.
D) Use the Lagrange Multiplier method to find the maxima and minima of f on the constraint curve $x^2 + y^2 = 9$.
E) What are the biggest and smallest values of f on the region $\{(x, y) : x^2 + y^2 \leq 9, x \geq 0, y \geq 0\}$?

II.
A) At the corners, the midpoints of the sides, and the center of the rectangle $R = [1, 3] \times [0, 2]$ in the xy-plane, the values of a function $f(x, y)$ are measured to be

$$\begin{array}{ccc} f(1,2) = 6 & f(2,2) = 8 & f(3,2) = 10 \\ f(1,1) = 3 & f(2,1) = 5 & f(3,1) = 7 \\ f(1,0) = 2 & f(2,0) = 4 & f(3,0) = 6 \end{array}$$

Estimate $\int\int_R f(x, y)\, dA$ using an appropriate Riemann sum.
B) The function $f(x, y)$ from part A was $f(x, y) = 2x + y^2$. Compute $\int\int_R f(x, y)\, dA$ exactly.
C) An solid object has "shadow" equal to R from parts A and B in the xy-plane. It is bounded above by the surface $z = 3x^2 + y^2 e^y$ and below by the surface $z = 2\cos(\pi(x)) + y^2$. Find its volume.
D) What is the x-coordinate of the center of mass of the volume from part C if the mass density at each point is a constant c?

III. A region S in the first and fourth quadrants of the xy-plane is bounded by the lines $y = x, y = -x$, and by an arc of the circle $x^2 + y^2 = 9$.
A) Find the limits of integration to set up $\int\int_S y\, dA$ as one or more iterated integrals of the form $\int\int y\, dydx$. Evaluate.
B) Now find the limits of integration to set up $\int\int_S y\, dA$ as one or more integrals of the form $\int\int y\, dxdy$. Evaluate
C) Find the limits of integration to express $\int\int_S y\, dA$ in polar coordinates. Evaluate.

D) Your three answers to parts A,B,C should be equal. What theorems tell you that must be the case?

IV. A region in $\mathbf{R}^3$ R is bounded defined by

$$R = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1, 4x^2 + 4y^2 \geq 1\}$$

Find the volume of R.