Mathematics 241 – Multivariable Calculus
Solution for Quiz 7 – November 9, 2007

(10) Find the critical points of the function $f(x, y) = 3x+2y$ on the constraint set $g(x, y) = x^2 + y^2 - 4 = 0$ by the method of Lagrange multipliers.

Solution: The Lagrange multiplier equations are

$$\begin{aligned} \frac{\partial f}{\partial x} &= \lambda \frac{\partial g}{\partial x} \\ \frac{\partial f}{\partial y} &= \lambda \frac{\partial g}{\partial y} \\ g(x, y) &= 0 \end{aligned}$$

or

$$\begin{aligned} 3 &= 2\lambda x \\ 2 &= 2\lambda y \\ x^2 + y^2 - 4 &= 0. \end{aligned}$$

From the first two equations,

$$3y = 2\lambda xy = 2x,$$

so we can eliminate λ to obtain:

$$y = \frac{2x}{3}.$$

Substitute this into the constraint equation to eliminate y:

$$x^2 + \left(\frac{2x}{3}\right)^2 = x^2 + \frac{4x^2}{9} = \frac{13x^2}{9} = 4.$$

This says $x^2 = \frac{36}{13}$, so $x = \pm\frac{6}{\sqrt{13}}$. Then from the equation $y = \frac{2x}{3}$, we get the corresponding values of y, $y = \pm\frac{4}{\sqrt{13}}$. Note that y must have the same sign as x, so there are only *two* critical points here:

$$\left(\frac{6}{\sqrt{13}}, \frac{4}{\sqrt{13}}\right), \quad \left(-\frac{6}{\sqrt{13}}, -\frac{4}{\sqrt{13}}\right).$$