MONT 104N – Modeling the Environment
Solutions for Midterm Exam – October 26, 2012

I. The following table gives amounts of carbon (in units of 10^{15} kilograms) contained in the major "reservoirs" of this element on planet Earth:

Reservoir	**Carbon Content**
Atmosphere	.59
Crust (as fossil fuels)	3.7
Vegetation	2.3
Shallow Ocean	.9
Deep Ocean	37.3

A. (15) What percent of the total carbon present on Earth is contained in each of these reservoirs?

Solution: The sum of these amounts is 44.79×10^{15} kilograms. So, the percentage of the carbon in the deep ocean reservoir, for instance, is

$$\frac{37.3}{44.79} \times 100\% \doteq 83.3\%$$

The other values are computed similarly. Rounding to 1 decimal place,

Reservoir	**Percent of Total Carbon**
Atmosphere	1.3%
Crust (as fossil fuels)	8.3%
Vegetation	5.1%
Shallow Ocean	2.0%
Deep Ocean	83.3%

B. (15) Construct and draw a chart (your choice of type) showing how the total carbon breaks down into these categories.

Solution: Either a pie chart or a bar chart is OK here. If you use a pie chart, the percents above correspond to the fractions of the pie. If you use a bar chart, it would be best to make the vertical axis scale correspond to the actual amounts of carbon (in units of 10^{15} kg), so use something like a vertical scale of 0 to 40.

II. Burning coal provides between 9500 and 14000 BTUs of heat energy per pound. Using the table below, answer questions A and B.

A. (10) Express the heat energy from burning one pound of coal as a range of values in calories.

Solution: There are 251.996 calories in one BTU. Hence, burning one pound of coal yields between

$$9500\ BTU \times 251.996\ \frac{cal}{BTU} \doteq 2.39 \times 10^6\ cal$$

and

$$14000\ BTU \times 251.996\ \frac{cal}{BTU} \doteq 3.53 \times 10^6\ cal.$$

B. (10) What is the range of heat energy values provided by burning 1 kilogram of coal, expressed in BTUs?

Solution: From the table, one pound is .4536 kg, so 1 kg = $\frac{1}{.4536} \doteq 2.20$ lb. Hence burning 1kg of coal provides between

$$9500\ BTU/lb \times 2.20\ lb = 2.09 \times 10^4\ BTU$$

and

$$14000\ BTU/lb \times 2.20\ lb = 3.08 \times 10^4\ BTU.$$

IV. (10) The 2012 Honda Civic comes in a standard (gasoline engine) version and a hybrid (gasoline-electric) version. The standard version has a fuel efficiency of about 34 miles per gallon, while the hybrid version gets about 44 miles per gallon. The hybrid version has a price about $4000 more than the standard. If you buy the hybrid and drive about 10000 miles per year, about how many years will it take for your savings in gasoline costs to make up for the difference in price? Assume that gasoline will average $4.00 per gallon over the life of your vehicle.

Solution: If you drive 10000 miles with the hybrid version, then you use $10000 \times \frac{1}{44} \doteq 227.3$ gallons of gas in one year and you spend $227.3 \times \$4.00 \doteq \$909.09$ on gasoline. With the standard version you use $10000 \times \frac{1}{34} \doteq 294.1$ gallons of gasoline and you spend $1176.47. The difference is $267.38. To make up the $4000 greater cost of the hybrid model, you would need to keep the hybrid for about $\frac{4000}{267.38} \doteq 15$ years. (Comment: These are pretty realistic numbers, and this is one reason why hybrids are still seen as less attractive options by many people.)

IV. In 2002, there were about 4.71×10^5 alternate fuel vehicles (powered by electricity, ethanol, natural gas, etc.) on the road in the U.S. By 2004, the number had risen to 5.48×10^5.

A. (5) What was the percent change in alternate fuel vehicles from 2002 to 2004?

Solution: The percent change was

$$\frac{5.48 \times 10^5 - 4.71 \times 10^5}{4.71 \times 10^5} \times 100\% \doteq 16.3\%$$

B. (6) Construct a linear model for AFV = number of alternate fuel vehicles as a function of t = time in years using the information above. Take $t = 0$ corresponding to 2002. What does your model predict about the number AFV in 2011?

Solution: With $t = 0$ corresponding to 2002, from the point-slope form we have the linear model

$$AFV = \frac{5.48 - 4.71}{2} t + 4.71 = .385t + 4.71$$

(The units of AFV here are 10^5 vehicles.) If $t = 0$ corresponds to 2002, then 2011 corresponds to $t = 9$, so the model predicts

$$AFV = (.385)(9) + 4.71 \doteq 8.2$$

(times 10^5) alternate fuel vehicles.

C. The following spreadsheet shows a scatterplot for the number of AFV vehicles on the road (in units of 10^5 vehicles) over a longer time period starting in 1995. Place a check next to the correct responses to the questions below.

1. (3) The least squares regression line for this data set would have a slope m about:
$m = -.1$ __________
$m = .3$ ____ *X* ____
$m = 1$ __________

The best answer is about $m = .3$ since AFV changes by about 3 units over the 9 years. The change is definitely not negative so $m < 0$ does not make sense, and the slope is definitely not as big as 1.

2. (3) Based on the scatter plot, the correlation coefficient r is probably:
between .5 and 1 ____ *X* ____
between 0 and .5 __________
between $-.5$ and 0 __________
between -1 and $-.5$ __________

The best answer is $.5 < r < 1$ since the data is increasing (so $r > 0$) and relatively linear (so r is closer to 1 than to 0).

3. (3) Suppose you had used the two data points for 1999 and 2000 to construct a linear model and predicted an AFV number for 2004 from that. Which would be larger:
the actual value for AFV in 2004 __________
the predicted value for AFV in 2004 from the model ____ *X* ____

The line through those two points has much larger slope than the prevailing slope. So the predicted value would be much larger than the actual value in 2004.

Essay (30)

In group project 3, we studied the flow diagram of the U.S. energy economy in 2010 (see sheet at end of exam). Of the five end-use sectors of the economy given in that diagram, which use energy in the *most wasteful* ways? Which sectors are *more efficient* in their energy use? Propose and discuss some likely reasons why these differences in efficiency exist. Are there obvious changes that could be made to improve the efficiency of the most wasteful sectors? Which of the energy sources listed in that diagram are from "fossil fuels" and which are renewable resources? Is there a clear difference in the efficiency of use of fossil fuels versus use of renewable resources?

Model response: Electricity generation and transportation are the most wasteful sectors, while the residential, commercial, and industrial sectors are more efficient. One way to quantify this is to compute the percentage of the energy inputs to each of the sectors that is put to useful purposes and the percentage of input that is wasted. For the electricity generation sector, of the 39.49Q of input, 12.71Q or 32%, is successfully transmitted, while 68% is wasted immediately in transmission (and some of the electricity is also wasted in the other end-use sectors). In the transportation sector, the situation is even worse. Only about 6.86Q of the 27.45Q of energy inputs (25%) is put to useful purposes, while 75% is wasted. The corresponding figures for the residential, commercial, and industrial sectors are all very close to 80% usefully employed and 20% wasted.

Part of the inefficiency of electricity generation is due to the technology itself. Much of the energy produced by power plants is lost in transmission over the wires used to transport it to where it is used. Most of the inefficiency of the transportation sector has been due to the fact that over the course of the 20th century there were relatively cheap and plentiful sources of petroleum readily available. So there was little incentive to develop more efficient gasoline engines or to develop other technologies for transportation as long as that was true. In addition, powerful energy companies (such as the large multinational oil companies) actually discouraged the development of more efficient alternative technologies for much of this time. Residential, commercial, and industrial uses likely tend to be more efficient since the true costs of energy are more directly felt by consumers and business owners in these cases. So there have been incentives to make energy use more efficient in these sectors through more efficient heating and insulation of buildings, better lighting technologies, and other more efficient appliances.

The easiest way to reduce waste would probably be to increase the efficiency of gasoline engines (the mpg) in cars and trucks so that less of the energy input from petroleum is wasted and more is put to useful purposes. Increasing the efficiency of electricity generation is more difficult; it would probably require decentralization and placement of more smaller power plants closer to homes and businesses where the electricity is used.

The energy sources that are "fossil fuels" are natural gas, petroleum, and coal. The renewable sources are solar, hydro, wind, geothermal, and biomass. It is not immediately clear whether there is a large difference in efficiency between the fossil fuels and the renewable sources. The major differences in efficiency come in the end uses, and several of the renewable sources (wind, solar, hydro) are being used primarily in inefficient sectors such as electricity generation.