College of the Holy Cross, Fall Semester, 2012
MONT 104N – Modeling the Environment
Thursday December 13, 8 AM

Your Name: _____

Instructions Please write your answers in the spaces provided on the following pages, and show work on the test itself. **For possible partial credit, you must show work**. Use the back of the preceding page if you need more space for scratch work.

Please do not write in the space below

Problem	Points/Poss
I	/ 50
II	/ 35
III	/ 25
IV	/ 30
Essay	/ 60
Total	/200

Have a peaceful and joyous holiday season!

I. Wind power has emerged as the fastest growing source of energy for electrical power generation in recent years. In 2004, the generating capacity of all wind turbines in use was about $47,600$ megawatts and the generating capacity was increasing at about 26.8% per year.

A. (10) The typical English unit of power is the horsepower. 1 horsepower = .0007457 megawatts. Convert $47,600$ megawatts to the equivalent number of horsepower.

B. (10) Using the information above, construct an exponential model for WP = wind power generation as a function of t = years since 2004. *Use units of* 10^4 *megawatts for* WP.

C. (15) Fill in the table of values for WP below with values predicted by your model for the years $2004 - 2011$. Round to 2 decimal places. In what year did WP reach approximately double the 2004 level?

$Year$	2004	2005	2006	2007	2008	2009	2010	2011
WP								

D. (10) How many years will it take for wind power generation to reach $320,000$ megawatts according to your model?

E. (5) The following graph (produced by the Global Wind Energy Council – GWEC) shows the actual global wind electrical power generation capacity (estimated via surveys of electrical power producers). How do the actual figures compare with your model values? Note: The vertical scale of the graph is in *gigawatts*. 1 gigawatt = 1000 megawatts.

II. According to the United Nations Food and Agriculture Organization, in 2000, forest area covered 4.038×10^9 hectares of the Earth's surface. The forest area in 2010 was 4.033×10^9 hectares. Assuming that the decrease in forest area is linear, and that it will continue at the same rate into the future, in this problem you will develop a linear model for the forest area $FA =$ (in units of 10^9 hectares) remaining as a function of $t =$ years since 2000.

A. (10) Determine the slope for the linear model of the forest area.

B. (10) What is the linear equation modeling the forest area as a function of $t =$ years since 2000.

C. (10) Use your equation to predict the amount of forest area that will remain in 2020.

D. (5) According to your model, in what year will the forest area reach 4.0×10^9 hectares?

III. Suppose that a population of fast-reproducing insects in an area has a natural growth rate of 7% per month from births and deaths, and that there is a net migration *loss* of 100 individuals per month.

A. (5) Which of the following difference equation models for $P(n)$ = population in month n fits the description above? (Place a check next to the correct one.)

1) _____ $P(n) = 7P(n-1) - 100$
2) _____ $P(n) = 1.07P(n-100)$
3) _____ $P(n) = 1.07P(n-1) - 100$
4) _____ $P(n) = 1.07P(n-1) + 100$

B. (10) Using an initial value $P(0) = 500$, determine the populations in months $1, 2, 3, 4, 5$ according to the model you picked in part A and record the values in the following table (round any decimal values to the nearest whole number)

n	0	1	2	3	4	5
$P(n)$	500					

C. (10) What happens to the population in the long run as n increase? Does it tend to a definite value? What is that value?

IV. Answer any three of the following four questions (only the best three will be counted if you answer more than three).

A. (10) What does the correlation coefficient r (or its square r^2) measure? How did we use it? Explain what it would mean, for instance if $r^2 = 1$ or $r^2 = 0$.

B. (10) If you are fitting a power law model to a data set (x_i, y_i) "by hand," you would start by transforming the data to $(X, Y) = (\log(x_i), \log(y_i))$. If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding power law model? (Assume the logarithms have base 10 as we discussed in class.)

C. (10) A population has unrestricted growth rate $r_{max} = .03$ and carrying capacity $K = 1000$. What is the corresponding logistic model?

D. (10) The following graph shows W = the world production of photovoltaic arrays (used for solar power generation) in units of "peak megawatts" (the power capacity they have). Between 1998 and 2007, what type of model would be most appropriate for describing how W is growing. Explain. *Look at the vertical axis scale carefully!*

V. Essay. (60) In general terms, what is a mathematical model? Describe what they are, how they are constructed, and how they are used. Give examples of two different types of mathematical models we have studied in this course. Next, *why* do we try to build mathematical models of aspects of the real world? Can *any* mathematical model be a completely accurate representation of some aspect of the natural world? As an example, why do scientists think it is important to understand how much CO_2 is present in the atmosphere? What tends to happen when CO_2 levels rise? Describe a key piece of evidence that suggests human activities might have changed atmospheric CO_2 levels over the past 50-200 years. Explain the case for saying the evidence points to that conclusion, and relate your answer to the results of modeling exercises we did in this class.

Essay (continued)