MONT 105N – Analyzing Environmental Data
Midterm Examination Solutions – April 12, 2013

I. A fertile but short-lived organism has monthly fertility and survival rates as given in this table:

Age group	A	B	C
Age (months)	$0-1$	$1-2$	$2-3$
Fertility rate	0	4	3
Survival rate	0.3	0.1	0.0

A) (10) Draw the corresponding life-cycle graph.

Solution: The graph should have three circles marked A, B, C and arrows:

- from A to B marked 0.3
- from B to C marked 0.1
- from B to A marked 4
- from C to A marked 3.

B) (10) Convert the information in the table (and the life-cycle graph) to a system of difference equations.

Solution: The system is:

$$\begin{aligned} a(n) &= 4b(n-1) + 3c(n-1) \\ b(n) &= .3a(n-1) \\ c(n) &= .1b(n-1) \end{aligned}$$

C) (5) Given that $a(0) = 300$, $b(0) = 200$ and $c(0) = 100$, determine $a(1), b(1), c(1)$.

Solution: Using the difference equations from B) we have

$$\begin{aligned} a(1) &= 4b(0) + 3c(0) = 800 + 300 = 1100 \\ b(1) &= .3a(0) = 90 \\ c(1) &= .1b(0) = 20 \end{aligned}$$

II. All parts of this question refer to the following data set with $n = 9$:

$$12, 15, 18, 25, 26, 28, 31, 33, 34$$

A) (10) Find the "5-number summary" and draw the corresponding box plot.

Solution: We have Min = 12, $Q_1 = 18$, Median = 26, $Q_3 = 31$, and Max = 34. The box plot has box from 18 to 31, with a vertical line at 26, plus "whiskers" extending to 12 on the left and 34 on the right. (*Note:* I included the median in the upper and lower halves to do this. If you don't and take only the numbers strictly less and strictly larger than the median in the halves, then you get slightly different results. Those will also get full credit (assuming they are correct, of course!)

B) (5) Is the Bowley measure of skewness positive or negative for this data set?

We have

$$\text{skewness} = \frac{31 - 2 \cdot 26 + 18}{31 - 18} = \frac{-3}{13} < 0.$$

This should look right given the shape of the box plot.

C) (5) Given that the SD is approximately $SD = 7.97$, how well does the Empirical Rule "fit" this data? (Say what that rule says and then determine how closely this data follows it.)

Solution: The Empirical Rule says that (if the data were normally distributed), then about 68% of the data values would be within one standard deviation of the mean, about 95% of the data values would be within two standard deviations of the mean, and almost all, or 99.7% of the data values would be within three standard deviations of the mean. Here if we add the numbers x_i and divide by $N = 9$, we get $\overline{x} \doteq 24.67$. So the interval $\overline{x} - SD$ to $\overline{x} + SD$ is 16.70 to 32.64. Five out of nine, or 55% of the data values are in this range. The interval within 2 SD's of the mean is $\overline{x} - 2SD = 8.73$ to $\overline{x} + 2SD = 40.61$. All nine, or 100% of the data values are in that range, hence also within 3 SD's of the mean. This is not too close to the Empirical Rule, but that is not unusual for such a small collection of data, especially given the skewness.

III. Suppose that a data set of $n = 25$ Dungeness crab shell length measurements has been collected. The mean shell width in the sample is $\overline{x} = 20$cm and the SD is 4cm.

A) (5) Would the z-score of a measurement $x = 18$ be?

Solution: The z-score would be

$$z = \frac{18 - 20}{4} = -.5$$

B) (5) What width measurement would correspond to a z-score of -1.3?

Solution: If $-1.3 = \frac{x-20}{4}$, then $x = 20 - (1.3)(4) = 14.8$cm.

C) (5) Find a 95% confidence interval for the population mean shell width.

Solution: Using the entry for $N = 25$ from the t-table, the confidence interval is

$$\mu = \overline{x} \pm t\frac{SD}{\sqrt{N}} = 20 \pm (2.064)\frac{4}{\sqrt{25}} \doteq 20 \pm 1.65.$$

(Note: This computation assumes that the whole population of Dungeness crab width measurements has a normal distribution.)

D) (10) Estimate the proportion of the population of crabs with shell widths between 18 and 21cm. State any assumptions you are making to arrive at your estimate.

Solution: Assuming that the whole population of Dungeness crab width measurements has a normal distribution, this proportion can be estimated as the area under the standard normal curve between $z = \frac{18-20}{4} = -.5$ and $z = \frac{21-20}{4} = .25$. From the symmetry of the normal curve, this is equal to the area between 0 and 0.5, plus the area between 0 and 0.25. The table then gives the value

$$.0987 + .1915 = .2902$$

In other words, about 29% of all crabs have shell widths between 18cm and 21cm.

IV. Essay. (30) Exactly what is the "hockey stick" graph? How was it originally generated? When and where did it first appear in the published scientific record? Have other studies confirmed this general pattern or called into question the original conclusions? When and why did the "hockey stick" become such a contentious issue in the climate change debate? What is the IPCC and what do they do? What was the role of the IPCC here?

Model Answer: (I will give full credit for less specific and detailed answers but here's a really complete answer) The original "hockey stick" graph is a plot showing the difference between reconstructed northern hemisphere average temperatures over the period from 1000 CE to 1800 CE and the actual 1961 CE to 1990 CE average temperature, together with the difference between actual average temperatures in recent years and the same 1961 CE to 1990 CE average. The reconstructed temperatures from the period before actual temperature measurements were being taken were computed from proxy data including tree ring widths, ice core samples, and coral growth measurements. The reconstructions used a relatively sophisticated statistical technique (called *principal components analysis*) that allowed the estimation of error bars as well as the mean temperature estimates. The "point" of the graph is that it seems to show a steep increase in recent years (coinciding with the period where human-produced greenhouse gas emissions have also reached high levels). The current average temperature measurements give values higher even than the maximum values in the error bars in all the years before 1800. A similar plot over a shorter timescale, and the techniques used to produce it, were published in the paper "Global Scale Temperature Patterns and Climate Forcing Over the Past Six Centuries" by Michael Mann, Raymond Bradley, and Malcolm Hughes from *Nature* in 1998. (The "standard version" of the graph itself originally appeared in a paper by the same authors titled "Northern Hemisphere Temperatures During the Past Millennium: Inferences, Uncertainties, and Limitations" that appeared in *Geophysical Research Letters* in 1999.) There have been many similar studies since 1998 that have confirmed Mann, Bradley, and Hughes' conclusions (at least in broad terms, possibly with some variations or different interpretations in some cases). Some other published work has questioned their conclusions, but most of that has been found to contain errors of various sorts, so the scientific consensus is strong that Mann, Bradley, and Hughes' general conclusions are correct. A version of the "hockey stick" graph was included in the Third Assessment Report of the IPCC (Intergovernmental Panel on Climate Change) in 2001. The IPCC is a scientific group established by the United Nations (through the World Meteorological Organization). Through its periodic Assessment Reports, it advises the member nations of the UN about climate change issues. The "hockey stick" became very controversial after the 2001 IPCC report was published because it seemed to present very striking evidence that human-caused climate change was real. Many climate-change "skeptics" went to great lengths to try to create doubt about the Mann-Bradley-Hughes work in the public mind and hence delay action on reducing greenhouse gas emissions and other environmental initiatives designed to reduce the effects of climate change.