MONT 102N – Modeling The Environment
Solutions for Final Examination – December 14, 2011

I. In 1990, forests covered 4.047×10^9 hectares of the Earth's surface. By 2000, forest area had decreased to 4.038×10^9 hectares. Assuming that the decrease in forest area is linear, and that it will continue at the same rate into the future, in this problem you will develop a linear model for the forest area remaining as a function of t = years since 1990.

A. (10) Determine the slope for the linear model of the forest area.

Solution: The slope is

$$m = \frac{4.038 - 4.047}{2000 - 1990} = -.0009$$

(in units of 10^9 hectares per year; this is equivalent to a net loss of 900,000 hectares, or 9000 square kilometers per year).

B. (5) What is the linear equation modeling the forest area as a function of t = years since 1990.

Solution: Writing FA for the forest area, the model would be

$$FA = 4.047 - .0009t$$

C. (10) Use your equation to predict the amount of forest area that will remain in 2010.

Solution: 2010 is $t = 20$ years after 1990, so the model predicts

$$FA = 4.047 - (.0009)(20) = 4.029 \ (\times 10^9 \text{ hectares})$$

D. (10) According to your model, in what year will the forest area reach 4.0×10^6 hectares?

Solution: We solve for t from the equation:

$$4.0 = 4.047 - .0009t,$$

so

$$t = \frac{4.047 - 4.0}{.0009} \doteq 52$$

This corresponds to the year 2042.

E. (5) According to the United Nations Food and Agriculture Organization, the actual forest area remaining in 2010 was 4.033×10^6 hectares. Discuss the relation between your prediction in part C and the actual figure. What is apparently happening?

Solution: The rate of deforestation was slower between 2000 and 2010 than the rate between 1990 and 2010. So the model predicted a lower forest area remaining than the actual figure.

II. Wind power has emerged as the fastest growing source of energy for electrical power generation in recent years. In 2004, the generating power of wind turbines was about $40,000$ megawatts and it was increasing at about 25.99% per year.

A. (10) The typical English unit of power is the horsepower. 1 horsepower = .0007457 megawatts. Convert $40,000$ megawatts to the equivalent number of horsepower.

Solution: Thinking about the unit conversion, we see

$$\text{horsepower} = \text{megawatt} \times \frac{\text{horsepower}}{\text{megawatt}}$$

That is there are

$$\frac{1}{.0007457} \doteq 1341$$

horsepower in one megawatt. Hence $40,000$ megawatts is

$$40,000 \times \frac{1}{.0007457} = 5.364 \times 10^7 \text{ horsepower}$$

B. (10) Construct an exponential model for WP = wind power generation in units of 10^4 megawatts, as a function of t = years since 2004.

Solution: In units of 10^4 megawatts, the model equation is:

$$WP = (4.0)(1.2599)^t$$

C. (15) Fill in the table of values for WP below with values predicted by your model for the years $2004 - 2010$. Round to 2 decimal places. In what year did WP reach approximately double the 2004 level?

$Year$	2004	2005	2006	2007	2008	2009	2010
WP	4.00	5.04	6.35	8.00	10.08	12.70	16.00

The wind power generation doubled by 2007 (and again by 2010).

D. (10) How many years will it take for wind power generation to reach $320,000$ megawatts according to your model?

Solution: $320000 = 32 \times 10^4$, so the equation we want to solve is $32 = 4(1.2599)^t$. This is true when $t = \frac{log(8)}{\log(1.2599)} \doteq 9$, or in the year 2013. (This can also be seen without the calculation since the table shows that the multiplier $M = 1.2599$ gives $M^3 \doteq 2$. So the next doubling to 320000 will occur in another 3 years after 2010.)

III. Suppose that a population of fast-reproducing insects in an area has a natural growth rate of 5% per month from births and deaths, and that there is a net migration *loss* of 20 individuals per month.

A. (5) Which of the following difference equation models for $P(n)$ = population in month n fits the description above? (Place a check next to the correct one.)

1) ______ $P(n) = 5P(n-1) + 20$
2) ______ $P(n) = 1.05P(n-20)$
3) __X__ $P(n) = 1.05P(n-1) - 20$
4) ______ $P(n) = 1.05P(n-1) + 20$

B. (15) Using an initial value $P(0) = 500$, determine the populations in months $1, 2, 3, 4, 5$ according to the model you picked in part A and record the values in the following table (round any decimal values to the nearest whole number)

Solution:

n	0	1	2	3	4	5
$P(n)$	500	505	510	516	522	528

C. (5) What happens to the population in the long run as n increase? Does it tend to a definite value?

Solution: It seems that the population will continue increasing at an increasing rate as n increases. Note that if $P(n-1) > 400$, then $P(n) = (1.05)P(n-1) - 20 > P(n-1)$ and the difference $P(n) - P(n-1) = (.05)P(n-1) - 20$ is bigger the larger $P(n-1)$ is.

IV. Answer any two of the following three questions (only the best two will be counted if you answer more than two).

A. (15) If you are fitting a power law model to a data set (x_i, y_i) "by hand," you start by transforming the data to $(X, Y) = (log(x_i), log(y_i))$. If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding power law model? (Assume the logarithms are log with base 10 as we discussed in class).

Solution: The linear equation is equivalent to $\log(y) = m\log(x) + b$ in terms of the original variables. So exponentiating both sides we get $y = 10^b \cdot x^m$. In other words, m is the power in the power law, and 10^b is the constant multiplier.

B. (15) What does the correlation coefficient r (or its square r^2) measure? How did we use it? Explain what it would mean, for instance if $r = 1$ or $r = -1$.

Solution: The correlation coefficient r measures the degree of linearity in a scatter plot (in other words, how close the data points come to lying on a single straight line). If $r = 1$, then all the points are on a single line of positive slope. If $r = -1$, all the points are on a single line of negative slope. We used this to measure the goodness of fit even for exponential or power law models. When we did this, we were looking at the correlation coefficient for the *transformed* data (the $(\log(x_i), \log(y_i))$ in the power law case, and the $(x_i, \log(y_i))$ in the exponential case).

C. (15) What type of chart (scatterplot, pie chart, bar chart, etc.) would be most useful to describe the composition of a forest if there 5 different types of trees present in different concentrations per acre? Explain, and illustrate your answer with a chart if a typical acre of forest contains 10 oaks, 12 maples, 5 pines, 2 hemlocks, and 1 chestnut.

Solution: For a chart indicating the composition of a whole made up of several parts, either a pie chart or a bar chart could be used. But a *pie chart* would be a slightly superior choice to show the composition. For a pie chart, we would compute the percentages of the whole represented by each species: $10 + 12 + 5 + 2 + 1 = 30$ trees. So oaks account for $10/30 \times 100\% = 33.3\%$, maples account for $12/30 \times 100\%$, or 40%, pines account for $5/30 \times 100\%$, or 16.7%, hemlocks account for 6.7%, and chestnuts account for the remaining 3.3%. These would be shown as fractions of a whole circle in the pie chart.

V. Essay. (60) What are the major reservoirs of carbon in the Earth's short-term carbon cycle? What are the major flows ("fluxes") of carbon between those reservoirs. Describe them in words and in a diagram. In our final project, we looked at a simple model of this system. What features of the real-world carbon cycle did that model include? What are three possibly important features of the real-world carbon cycle that were left out?

Solution: A complete answer should include the following reservoirs: the atmosphere, the surface ocean, the deep ocean, the terrestrial biosphere (all living plants and animals on land), and soil/debris on land. The major fluxes are gas exchange between the atmosphere and surface ocean, downwelling, and the "biopump" from the surface ocean to the deep ocean, and upwelling from the deep ocean to the surface ocean. Up-and down-welling are physical mixing of the different layers of ocean water. In the "biopump" carbon from ocean plants and animals, including mostly plankton, sinks to sediments in the ocean floor. Then there are fluxes of photosynthesis from the atmosphere to the terrestrial biosphere, and respiration going the other way. Photosynthesis is the process by which plants take carbon dioxide out of the atmosphere and incorporate the carbon into their tissues. Oxygen is released in the process, which is breathed in by animals. Animal respiration releases carbon dioxide into the atmosphere. Death takes carbon from the biosphere to soil/debris, and decomposition takes carbon from the soil/debris reservoir to the atmosphere. Finally, human activity from fossil fuel burning puts carbon into the atmosphere, and deforestation takes carbon out the terrestrial biosphere and adds it to the atmosphere.

See the class notes for the associated diagram.

Some obvious things our model did not take into account were:

1. It did not account for the way increases in global temperature might affect the carbon fluxes in the cycle. Those rates were constant or constant proportions (multipliers).

2. As a climate model, it did not take the effects of other atmospheric gases such as water vapor, methane, etc. into account. Some of these (especially methane) might be even stronger greenhouse gases than carbon dioxide.

3. It did not take into account the fact that the total energy received by the Earth from solar radiation actually varies over time. This can cause variations in photosynthesis, carbon levels, and in climate as well.

4. Volcanoes add some CO_2 into the atmosphere when they erupt.

On the other hand, some human activities such as decreasing our rate of fossil fuel burning or deforestation *can* be accounted for in the model, and we investigated their effects in one the questions on the project. But no mathematical model can include *everything* that might affect the behavior of the real-world system that is being modeled.