MONT 103N – Analyzing Environmental Data
Solutions for Midterm Examination – March 30, 2012

I. In the absence of any natural predators, the population in month n, denoted $u(n)$, of a fast-reproducing species of fruit flies satisfies the logistic difference equation:

$$u(n) = 2.3u(n-1) - \frac{1.3}{300} \cdot (u(n-1))^2$$

A) (10) If $u(0) = 100$, what are $u(1)$ and $u(2)$?

Solution: We have

$$u(1) = 2.3 \cdot 100 - \frac{1.3}{300} \cdot (100)^2 \doteq 230 - 43.3 = 186.7$$

Then

$$u(2) = 2.3 \cdot 186.7 - \frac{1.3}{300} \cdot (186.7)^2 \doteq 278.3$$

B) (10) Now suppose that birds living in the same habitat kill and eat 50% of the fruit fly population present in each month. How could the model be adjusted to take this into account?

Solution: This would be incorporated by subtracting off a term on the right:

$$u(n) = 2.3u(n-1) - \frac{1.3}{300} \cdot u(n-1)^2 - .5 \cdot u(n-1)$$

This can also be simplified to

$$u(n) = 1.8u(n-1) - .004\overline{3} \cdot u(n-1)^2$$

C) (5) What are the equilibrium fruit fly populations for your adjusted model in part B?

Solution: Equilibrium levels of the population are solutions e of the equation

$$e = 1.8e - \frac{1.3}{300}e^2,$$

or

$$\frac{1.3}{300}e^2 = .8e$$

The solutions are $e = 0$ and $e \doteq 184.6$.

II. All parts of this question refer to the following data set:

$$17, 18, 19, 26, 27, 29, 38, 41, 48$$

A) (10) Find the "5-number summary" and draw the corresponding box plot.

Solution: The summary is: Min = 17, $Q_1 = 19$, Median = 27, $Q_3 = 38$, Max = 48. The box plot should show a box from 19 to 38 with the Median marked, and "whiskers" to 17 on the left and 48 on the right.

B) (5) Is the Bowley measure of skewness positive or negative for this data set?

Solution: The measure of skewness should be positive, since the data exhibits a skew toward the larger values. Computing we have

$$\text{skewness} = \frac{Q_3 - 2 \cdot \text{Median} + Q_1}{Q_3 - Q_1} = \frac{38 - 2 \cdot 27 + 19}{38 - 19} = \frac{3}{19} > 0$$

C) (5) Find the standard deviation (SD) of this data set.

Solution: We have $\overline{x} = \frac{17+18+19+26+27+29+38+41+48}{9} \doteq 29.22$. So then

$$SD = \sqrt{\frac{\sum(x_i - \overline{x})^2}{8}} \doteq 10.97$$

III. Suppose that a data set of $n = 100$ lobster tail length measurements has been collected. The mean shell length in the sample is $\overline{x} = 15$cm and the SD is 5cm.

A) (5) Would the z-score of a measurement $x = 12$ be?

Solution: If $x = 12$, $z = \frac{12-15}{5} = -.6$

B) (5) What length measurement would correspond to a z-score of 1.7?

Solution: If $z = 1.7$, then $x = 15 + 1.7 \cdot 5 = 23.5$cm.

C) (5) Find a 95% confidence interval for the population mean tail length.

Solution: Since $n = 100 > 30$, the margin of error is $1.96 \cdot \frac{5}{\sqrt{100}} = .98$ and the confidence interval is $15 \pm .98$, or $(14.02, 15.98)$.

D) (10) Estimate the proportion of the population of lobsters with tail lengths between 13 and 21cm. Are you assuming something here?

Solution: Assuming the underlying population has a normal distribution, we can do this by finding the area under the standard normal curve between the corresponding z-scores. $x = 13 \leftrightarrow z = -.4$ and $x = 21 \leftrightarrow z = 1.2$. The corresponding area under the standard normal curve is $.1554 + .3849 = .5403$

IV. Essay. (30) In Chapter 2 of *The Botany of Desire*, Michael Pollan says "the tulip is that rare figure of Apollonian beauty in a horticultural pantheon presided over by Dionysus," and "color breaks can perhaps best be understood as an explosive outbreak of the Dionysian in the too-strict Apollonian world of the tulip – and the Dutch bourgeoisie." Explain these statements: Who (or what) are Apollo and Dionysus? What do they represent? How did the figure of Dionysus appear in Chapter 1 as well?

Solution: In Greek mythology, Apollo was the sun god and he came to represent light, order, rationality, civilization, and restraint. On the other hand, Dionysus was the god of the grape harvest and he came to represent darkness, disorder, ritual madness, wildness, and the excess of drunkenness. So Apollo and Dionysus symbolize two fundamental desires that we can see at work in all humans and human societies – the desire for order and rationality and the desire for wildness and intoxication. In the first quotation, Pollan is comparing the pure, symmetrical, restrained (Apollonian) beauty of standard tulips with the wildness and (Dionysian) exuberance of form and color of flowers generally. The different colors, shapes, and textures exhibited by flowers can be said to have some of the wildness and exuberance of Dionysus. In the second quotation, within the specific realm of tulips, Pollan is pointing out the Dionysian aspects of the "broken" tulips such as the *Semper Augustus* and *Viceroy* varieties that were popular during the "tulipomania" period in the Netherlands in the 1630's. According to Pollan, the "broken" tulips were like an outbreak of the wild and disorderly Dionysus in that orderly, Apollonian world of tulips (and of Dutch society of the time). The "tulipomania" phase can be seen as a sort of Dionysian intoxication with those flowers. (In a way, when it is expressed openly, the Dionysian side can be even stronger in a society that prizes the Apollonian as much as the Dutch did.) The duality between Apollo and Dionysus is a consistent thread that runs throughout *The Botany of Desire.* In Chapter 1, for instance, the figure of John Chapman ("Johnny Appleseed" as he really was, not how the myths have portrayed him) is compared with Dionysus. This has a certain justice, after all, since Chapman's main reason for spreading apple trees around was to provide the raw material for the production of "hard" cider (the equivalent of wine for the American frontier).