al-Khwarizmi: The Inventor of Algebra

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Abu Jafar Muhammad ibn Mus al-Khwarizmi amassed mathematical knowledge from diverse cultures to develop the technique and practicality of Algebra, which in turn revolutionized mathematical thinking throughout the world. The egalitarian Islamic empire that was prominent prior to and during al-Khwarizmi’s lifetime encouraged toleration and diversity. This culture enabled ideas and manuscripts from different locations and different time periods to be exchanged, which accelerated the development of new ideas. Al-Khwarizmi was exposed to this wealth of knowledge in the library at the House of Wisdom, which enabled him to write the foundation of algebra in his “*Compendium on Calculation by Completion and Reduction*.” His texts demonstrated the new procedure of systemic algebra and emphasized its functionality within Islam culture. Al-Khwarizmi’s other texts improved the methods of practical mathematics regarding astronomy, numerals and geometry. The knowledge about algebra and its functionality spread throughout the Islamic Empire, which were adapted to many different problems. Eventually, al-Khwarizmi’s numerals and algebraic methods reached Europe, which transformed conventional mathematics. Therefore, al-Khwarizmi’s foundation of algebra established a new branch of mathematics that stressed practical application over academic usage.

The spread of the Islamic empire across the Arab world created an egalitarian culture that laid the foundation for the future exchange of mathematical ideas for al-Khwarizmi. In the year 622 AD Prophet Muhammad ignited the spread of Islam as he fled from Mecca and attracted many pilgrim followers.[[1]](#footnote-1) The passionate and egalitarian characteristics of Islam captured both the imagination and support of people, who were eager to escape the authority of local leaders[[2]](#footnote-2). The common historic roots between Muslims, Jews and Christians, explains the emphasis of religious tolerance within the Islam faith.[[3]](#footnote-3) The Umayyad dynasty drastically expanded the Islamic empire to multiple ethnic territories including northern Africa, Spain, and Turkey.[[4]](#footnote-4) Although religious tolerance was a core value of Islamic faith, the Umayyad dynasty discriminated and persecuted against non Muslims because they were focused on maintaining control over new territories.[[5]](#footnote-5) The spread of Islam within the Arabic world united many ethnic groups, which would ultimately be the resources of al-Khwarizmi’s mathematical influences. Despite the religious discrimination within the Umayyad dynasty, they successfully expanded the influence of Islam to encompass a multicultural landmass. Without the foundation of the Islamic culture and empire, al-Khwarizmi might not have had the beneficial environment to develop algebra.

The new Abbasid Dynasty encouraged a culture of diversity and tolerance that promoted the exchange of ideas; which provided a foundation of learning for al-Khwarizmi. In 750 AD the Umayyad Dynasty was overthrown, and the new Abbasid Empire allowed religious freedom within the many ethnically diverse areas of Islamic Empire.[[6]](#footnote-6) The new Muslim empire, “restored ancient ties among historic centers of civilization across a huge landmass,” and united cultures that had been separated because of political divisions. [[7]](#footnote-7) Therefore, the tolerant and diverse culture formed a melting pot of intellectual traditions: the Hellenistic learning of Greece and Alexandria, knowledge from Sumerian, Persian, and Indian cultures, as well as ideas from Christians, Jews, and Pagans.[[8]](#footnote-8) Al-Khwarizmi drew upon mathematic influences from each of these regions, thus the tolerance of the Abbasid Dynasty enabled the exchange of each regions respected sciences. The abundance of diverse knowledge being shared freely would develop a culture that would provide al-Khwarizmi with the resources necessary to inspire his mathematical ideas.

The establishment of the cultural city of Baghdad and the knowledge of papermaking, were critical steps that would unite volumes of mathematical manuscripts in a single location; consequently this was critical to provide al-Khwarizmi with mathematical texts. The Abbasid Dynasty needed to construct a multicultural capital city to anchor the diverse empire and to attract travelers and traders. In 752 AD the location of Baghdad was selected as the capital of the empire because of proximity to trade routes along the Tigris River and distance away from military threats.[[9]](#footnote-9) The city rapidly flourished as a vibrant multi-cultural center with a demand for trade, commerce and intellectual and scientific exchanges.[[10]](#footnote-10) Coincidentally the knowledge of papermaking acquired from Chinese prisoners in 751 AD was critical to the eventual growth of knowledge within the Islamic empire.[[11]](#footnote-11) The new inexpensive durable writing material accelerated the production and spread of manuscripts compared to what was possible with the traditional use of animal hides.[[12]](#footnote-12) Manuscripts written from different cultures within the Islamic empire were gathered within the multicultural capital of Baghdad. As a result, Baghdad became a center of intellectual learning and scientific inquiries. Al-Khwarizmi relied heavily on easy access to paper manuscripts from distant regions in order to ignite his revolutionary algebraic techniques. The location of Baghdad as a trade center brought together and dispersed mathematical texts to a single location where al-Khwarizmi could conveniently learn from.

As a result, The House of Wisdom in Baghdad was established to facilitate the exchange of wisdom within the Abbasid Dynasty, where al-Khwarizmi would eventually study. The House of Wisdom, which was made up of a translation bureau, a library academy of scholars, and intellects from across the empire, was constructed to safeguard the invaluable knowledge within the Islamic empire.[[13]](#footnote-13) Under the Abbasid Dynasty, scholars began to translate foreign classics written by Hindu, Persian and Greek Scholars into Arabic.[[14]](#footnote-14) Al-Khwarizmi relied on influences from diverse cultures to stimulate new ideas that previous mathematicians failed to discover. Over time the wealth of knowledge written into Arabic enabled it to become the new universal language for scientific inquiry.[[15]](#footnote-15) Therefore the convenient access to manuscripts translated into a single medium, which were preserved and safeguarded in one location, were instrumental as al-Khwarizmi studied mathematics from new regions.

Although little about al-Khwarizmi‘s early life is known, the Abbasid culture under al-Ma’mun enabled al-Khwarizmi to study at the House of Wisdom. Muhammad ibn Musa al-Khwarizmi was born around AD 780, and his family name “al-Khwarizmi” indicates that he or his family originated from Khwarizm, east of the Caspian Sea, in modern-day Uzbekistan.[[16]](#footnote-16) Evidence also suggests that al-Khwarizmi may have been of Zoroastrian descent, which indicates he could have learned about Indian mathematics and Zoroastrian astrology during his early life.[[17]](#footnote-17) It is possible that al-Khwarizmi converted from Zoroastrianism to Islam to benefit his social status.[[18]](#footnote-18) In 820 AD, al-Khwarizmi was appointed as a member of the House of Wisdom by Caliph al-Ma’mun, to whom he later dedicated his mathematical work.[[19]](#footnote-19) Al-Khwarizmi was influenced by mathematics from other cultures during his studies at the House of Wisdom, which inspired him to formulate the foundation of algebra. He worked in Baghdad until his death in 850, but little evidence remains about his daily life or family.[[20]](#footnote-20) Al-Khwarizmi’s exposure to many diverse cultures throughout his life might have prepared him to learn and analyze different mathematics to invent algebra.

Similar to Arabic sciences, al-Khwarizmi developed the theory of algebra to solve practical problems of Islamic society. He wrote *Hisab al-jabr w’al-muqabala*, or “*Compendium on Calculation by Completion and Reduction*” some time before 825 AD, and dedicated it to the Caliph al-Ma’mun.[[21]](#footnote-21) His book consisted of three sections, theoretical, mensuration, and legacies. Al-Khwarizmi’s purpose for writing this book was to improve practical mathematical methods. His primary reason was to make solving problems of legacies, in accordance with the Islamic law, as easy as possible. He mentions in his work,

“[It] has encouraged me to compose a short work on Calculating by (the rules of) Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealing with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds are concerned.” [[22]](#footnote-22)

al-Khwarizmi’s time at the House of Wisdom allowed him to improve many previous methods of practical mathematics to pioneer a new and more efficient way of problem solving, known today as algebra.

The first section of *Hisab al-jabr w’al-muqabala* explains the theoretical basis of algebra. Al-Khwarizmi used generalization of arithmetic using variables in equations to establish the foundation of algebra in order to solve practical Islamic mathematic problems rather than solely relying on geometric solutions. He first begins by saying,

“When I considered what people generally want in calculation, I found that it always is a number.”[[23]](#footnote-23)

His reference to the desire of people, in finding a number, indicates how his work was gauged on improving practical methods of mathematics. He then introduces what numbers are, followed by the introduction and elucidation of linear and quadratic equations. As shown in *The Crest of the Peacock*, al-Khwarizmi demonstrated that all possible linear and quadratic equation can be simplified into six different types;

Roots equal squares: bx = ax2 Roots equal numbers: bx = c Squares equal numbers: ax2 = c Squares and roots equal numbers: ax2 + bx = c Roots and numbers equal squares: bx + c = ax2 Squares and numbers equal roots: ax2 + c = bx

where *a, b,* c are all positive integers while x represents the “root”. He also provided geometrical solutions to each of his six different equations to show the validity of his work. Modern notations of his six equations have been provided because “Al-Khwarizmi uses no symbols or written equations throughout his work. Every mathematical process is expressed in words.”[[24]](#footnote-24) Al-Khwarizmi used traditionally accepted techniques of geometrical proofs to prove the functionality and validity of his new algebraic techniques. This probably allowed mathematicians, who were well versed in geometry, to quickly comprehend the procedure of algebra.

Through his time at the House of Wisdom al-Khwarizmi wrote the first systematic way of solving mathematical problems that was more efficient than previous solutions. In *The Crest of the Peacock,* George Joseph explains the significance of al-Khwarizmi’s title *Hisab al-jabr w’al-muqabala.*

“There were two meanings associated with *al-jabr*. The more common was ‘restoration,’ as applied to the operation of adding equal terms to both sides of an equation so as to remove negative quantities, or to ‘restore’ a quantity that is subtracted from one side by adding it to the other…… [On the other hand,] The common meaning of *al-muqabala* is the ‘reduction’ of positive quantities in an equation by subtracting equal quantities from both sides.”[[25]](#footnote-25)

The final part of the first section was titled “On Business Transactions”, where he introduced his practical methods for solving daily problems. Here is an example where al-Khwarizmi gives a problem and how to calculate its solution;

“A quantity: I multiplied a third of it and a *dirham* by a fourth of it and a dirham; it becomes twenty”[[26]](#footnote-26)

In modern terms, this is written as ( where a *dirham* is the quantity 1. After this al-Khwarizmi uses *al-jabr* and *al-muqabala* to solve the problem and this is what he said;

“Its computation is that you multiply a third of something by a fourth of something: it comes to a half of a sixth of a square. And you multiply a dirham by a third of something: it comes to a third of something; and [you multiply] a dirham by a fourth of something to get a fourth of something; and [you multiply] a dirham by a dirham to get a dirham … Thus its total, [namely] a half of a sixth of a square and third of something and a quarter of something and a dirham, is equal to twenty dirhams.”[[27]](#footnote-27)

In modern terms this can be written as . What he did here is what is commonly known as the foil method. After this step he used the new procedure of algebra to solve for x, (which he refers to as “something”) which is the root. This example shows how al-Khwarizmi was committed to the practical uses of mathematics. He uses the term *dirham* in his example*,* which was the currency used by the Islamic people, to better relate the example of algebra to the common people. In this section, al-Khwarizmi also demonstrated the rules of multiplication with numbers, even those that were unknowns, for instance x. By stressing the relationship between analytic and geometric solutions to such problems and introducing the decimal place system, al-Khwarizmi through his algebra, for the first time in math history, established the art of analysis as a worthy discipline in its own right and put in on an equal footing with the more glamorous geometry.[[28]](#footnote-28)

To further introduce an efficient way of practical calculations, the second section of al-Khwarizmi’s book on mensuration demonstrated a practical use for the newly developed theory of algebra on measurements. His revolutionary ideas were unlike prior methods to solving problems; he introduced his new mathematics in a very comprehensible manner which gained acceptance quickly. In this section al-Khwarizmi demonstrated how to calculate the areas and volumes of various objects.

“Al-Khwarizmi gives equations for finding the area and circumference of a circle, for example, and for computing the volumes of cones, pyramids, and truncated pyramids. At one point, he gives a fairly accurate estimate of the number π. Al-Khwarizmi also presents the famous Pythagorean theorem established by the Greeks: [*a2* + *b2* = *c2*]”[[29]](#footnote-29)

The reference of al-Khwarizmi giving equations to find the volumes and areas of many shapes shows how practical and simple his new mathematical methods were.

The theory of algebra was developed primarily to solve daily practical problems that were experienced. Al-Khwarizmi’s third and longest section was on legacies and inheritance. In Islam the *Qur’an* distinguished how inheritance was divided among immediate family members. But it was still a difficult task to calculate exactly how much each family member got because of varying ratios. It became even more difficult when the deceased had bestowed some of his/her land to a stranger, because by law, the stranger is to be paid before all other heirs. Furthermore the stranger is allowed a maximum of one third of the estate unless the heirs have allowed permission for a higher share.[[30]](#footnote-30) In the case where the deceased has some debt left, it became tremendously difficult to calculate inheritance to family members because the debt had to be compensated from the legacy. One problem in *Hisab al-jabr w’al-muqabala* is stated as follows;

“A man dies leaving his sons behind him, and bequeathing one-third of his capital to a stranger. He leaves ten dirhams of property and a claim of ten dirhams upon one of the sons.”[[31]](#footnote-31)

Although the solution of this problem well not be discussed here. This problem shows the complexities that arose after a beloved’s death in the Islamic society, where Islamic laws of inheritance had to be followed to partition the estate of the deceased. Due to al-Khwarizmi’s revolutionizing work, he made these practical problems easier to solve by giving a clear and efficient method.

Although algebra remains al-Khwarizmi’s most influential work, he also worked as an astronomer and geographer, which illustrates his emphasis on practical mathematics. Similar to the development of algebra, al-Khwarizmi used mathematics to solve practical problems within the Islamic culture. His famous work called the *Zij al-Sindhind*, which means “an astronomical handbook according to the *Sindhind*,” consisted of critical mathematical thinking even though it did not involve algebra. His *Zij* had a significant practical implication because the Islamic society required precise astronomical calendars to accommodate for religious events. In addition to that, calculating horoscopes for the nobles was another occupation for an astronomer like al-Khwarizmi.[[32]](#footnote-32) Furthermore, al-Khwarizmi was a geographer as well. The practical significance of his geography was that he was able develop an accurate map of Asia and Africa. Determining the direction of Mecca was an essential part of a Muslim’s daily life and al-Khwarizmi was able, using mathematics, to find the direction of Mecca for prayer accurately. The significance of both the *Zij* and his work in geography provided al-Khwarizmi vital information to solve daily problems within the Islamic culture, similar to problems of inheritance within algebra.

In his *Hisab al-jabr w’al-muqabala* al-Khwarizmi demonstrated some important properties of numbers, for instance that all numbers are derived from one. He uses this notion to continue his work in his second most famous work called “Treatise on Hindu Reckoning” which introduced Hindu numerals. This work is most commonly referred to its Latin name, *Algorithmi de numero indorum,* because there is no concise Arabic name. He wrote this in 825 AD and was the first Islamic mathematician to write about Hindu numerals despite their introduction into Baghdad around 773 AD.[[33]](#footnote-33) Al-Khwarizmi introduced Hindu numerals into Islamic mathematics to show their practical use and how it made arithmetic as easy as possible. He had introduced the theory of algebra to make practical mathematics easy, and he wanted to introduce a numeral system that was of high efficiency so that the two could complement each other and make daily use of practical mathematics and arithmetic as simple as possible. He says in his work;

“Concerning the numbering of the Indians by means of IX symbols in their universal system of numbering, for the sake of its ease and brevity, so that this work, to be sure, might as the smallest, and whatever there is in its as result of multiplication and division, also addition and subtraction, etc.”[[34]](#footnote-34)

He introduced [1,2,3,4,5,6,7,8,9,0] using traditional notation and this was the first time a place holder was presented in Arabic. He showed zero’s ability to make writing large numbers significantly easier in relationship to previous methods.[[35]](#footnote-35) In addition to introducing the Hindu numerals, al-Khwarizmi showed how to add, subtract, multiply, and divide with these numerals. He also demonstrated the multiplication and division of fractions.[[36]](#footnote-36) Al-Khwarizmi was the first person to demonstrate a place value system, but did not introduce decimal fractions.[[37]](#footnote-37) His treatise supplemented the techniques of algebra which illustrated the practical usage to solve problems. Together these developments revolutionized European methods of mathematics as they spread from Islamic borders into the European world.

However, many modern scholars question the originality of al-Khwarizmi’s algebra based on his exposure to larger numbers of ancient texts at the House of Wisdom. Scholars believe al-Khwarizmi might have organized the techniques of algebra based on his exposure to algebra texts from other cultures, and did not invent it. Some scholars suggest that al-Khwarizmi’s geometrical solutions were based from the mathematical texts of Book II of Euclid’s “Elements.” [[38]](#footnote-38) Researchers well studied in classical Greek mathematics believe that the first ten proofs in Book II were proved independent of each other, which illustrates “the power of the method of geometrical algebra.”[[39]](#footnote-39) Therefore, they argue that al-Khwarizmi’s exposure to the ancient texts of Greece, already illustrated algebraic thinking. Although al-Khwarizmi could have been exposed to the ancient translations of Greek mathematics, there lacks evidence that his work on algebra was directly influenced by them. Mathematical researches reveal that al-Khwarizmi’s techniques for geometrical solutions prove that he “remained outside the sphere of Greek influence.”[[40]](#footnote-40) Thus, there remains no evidence to justify that al-Khwarizmi’s algebra techniques were copied from classic Greek texts.

Additionally, scholars speculate that al-Khwarizmi’s geometrical and analytical approaches to algebra could possibly have been derived from a Mesopotamian origin, thus disproving al-Khwarizmi as the first inventor of Algebra. Although no direct evidence exists, mathematicians argue that a hypothetical connection exists between Babylonian equations that used geometry and al-Khwarizmi’s second degree equations.[[41]](#footnote-41) Therefore, scholars believe the analytic nature that composed Babylonian equations relates to the methods that al-Khwarizmi utilized to complete his study on algebra. Conversely, scholars also claim that the unique algebra of al-Khwarizmi was self sufficient in explaining the skepticisms that arose from similar quadratic equations.[[42]](#footnote-42) Scholars have already ignored a possible relationship between al-Khwarizmi’s algebra and Indian roots because of distinct contrasts of a quadratic base.[[43]](#footnote-43) Although scholars argue a connection between al-Khwarizmi and Mesopotamian roots, only hypothetical theories link Mesopotamian origins to al-Khwarizmi’s algebra.

Al-Khwarizmi was influenced by the mathematical knowledge at the House of Wisdom, but he independently possessed the creativity and genesis to fuse diverse aspects from multiple cultures of mathematics to establish the foundation of algebra. Although researchers argue the derivative of algebraic thinking, there exists little denial that al-Khwarizmi’s algebra represents “the first systematic treatment of the general subject of algebra as distinct from the theory of numbers”.[[44]](#footnote-44) According to Lyons, Arabic scientists were not relying on translations for ancient texts to advance the development of Arabic science, mathematics and philosophy. Conversely, the growth of organized research and gradual advances in these areas of Arabic study generated demand for more comprehensive translations from ancient and foreign texts. A discovery within mathematics would return Islamic scholars back to ancient Greek texts, which were then translated, edited and improved.[[45]](#footnote-45) Thus Islamic mathematicians and scientists emphasized mathematical progress instead of originality. Al-Khwarizmi was influenced by elements of Greek, Indian, local Islamic mathematicians, but al-Khwarizmi’s genius lay behind his ability to bring together mathematical aspects from different cultures and stress their importance.[[46]](#footnote-46) For example, in 825 AD al-Khwarizmi wrote the first text about Hindu numerals to illustrate their significance despite their arrival in Baghdad around 773 AD.[[47]](#footnote-47) Scholars believe that al-Khwarizmi ‘s algebra came as a result of welding the elements from Babylonian and Greek algebraic thinking.[[48]](#footnote-48) Although many mathematicians might have had the opportunity, al-Khwarizmi was the first to communicate the systematic treatment of algebra. Therefore his influences from other culture should not discredit his clever ingenuity to establish the foundation of algebra.

Mathematicians, such as Abu Kamil ibn Aslam, copied directly from al-Khwarizmi’s work which illustrates the influence and precedent of his ideas . Kamil wrote his own text titled, *Kitab fi al-jabr wa’l-muqabala*, which was directly based on al-Khwarizmi’s previous work. In his work, Kamil not only directly quoted from al-Khwarizmi but he also copied almost half of Khwarizmi’s original examples and solutions.[[49]](#footnote-49) Kamil was the successor of al-Khwarizmi who used Khwarizmi’s algebraic base along with the underpinnings of Greek mathematics to “create an algebra based on practical realities.”[[50]](#footnote-50) As previously noted, there existed little sense of originality within Islamic culture, so mathematicians often quoted al-Khwarizmi. Therefore al-Khwarizmi’s text established a precedent for algebra, which inspired many future mathematicians.

Al-Khwarizmi heavily influenced the development of mathematics across various cultures. His development of algebra revolutionized mathematics, because it improved existing cultures’ ability to solve practical problems. The dispersal of, al-Khwarizmi’s texts across the Islamic empire influenced regions and many cultures, and eventually reached Europe through Spain. His revolutionary mathematical thinking also allowed for his astronomical theories and work with the Hindu-Arabic numerals to be highly influential to many cultures. Prominent mathematicians also realized the importance of these numerals because of their facility and practicality.

Through the spread of al-Khwarizmi’s algebraic rhetorical techniques in Islamic empire, Abu’l al qalasadi adopted new symbolism to simplify the algebraic procedure. While studying in Spain during the 1400s, Abu’l al qalasadi took shortened and abbreviated Arabic words to represent the procedure of algebra. The shorten words were: “*wa*” for addition, “*illa*” for subtraction, “*fi*” for multiplication, and “’*ala*” for division.[[51]](#footnote-51) The new abbreviation indicates a sense of symbolism that would later evolve into simplified modern mathematical notation. Therefore mathematicians adapted and improved the techniques of algebra to make solving equations even easier. According to *Aladdin’s Lamp*, “It was from this work that Europe later learned the branch of mathematics known as algebra.”[[52]](#footnote-52) As evident from Abu’l al qalasadi, European and modern algebraic methods originate from al-Khwarizmi’s revolutionary description in “*Compendium on Calculation by Completion and Reduction*.”

Another example of the facility of al-Khwarizmi’s ideas was the rule titled *Hisab al-Khataayn* or the “*Rule of Double False Position*”. It became the popular method when solutions to linear equations were difficult under traditional methods. The rule was brought to Europe by the Arabs and is found in the works of ninth century Islamic mathematician al-Khwarizmi.[[53]](#footnote-53) The rule states, let’s find an unknown quantity x in a linear equation ax + b = 0. Let G1 and G2 be initial (incorrect) guesses for the value of x, and F1 and F2 be the errors of the guesses. So, a\*G1 + b = F1 (1) and a\*G2 + b = F2 (1). Therefore a\*(G1-G2) = F1-F2 (2) The first equations (1) are multiplied by G2 and the second (2) multiplied by G1. Subtract the two resulting equations one from the other and the result yields b\*(G2-G1) = (F1\*G2) - (F2\*G1) (3). Equation (3) is then divided by equation (2), the resulting equation is x = ((F1\*G2)(F2\*G1))/(F1-F2) (4).[[54]](#footnote-54)From Al-Khwarizmi’s algebraic texts, the rule of double false position made difficult linear equations easier to solve and more efficient.

Al-Khwarizmi not only revolutionized the world of mathematics with theoretical algebra, he also wrote a treatise on Hindu numerals that would comprise all modern numbers. “The Hindu Art of Reckoning”, describes the Hindu-Arabic numerals which would eventually become “the digits of the western world.”[[55]](#footnote-55) Europe adopted these Hindu-Arabic numerals as they were used in Spain, however, they were not initially accepted as quickly as his algebraic techniques. Christian Europe refrained from using the Indian numerals because they believed it could be easily confused with other numbers like 6 and 9. Also, Europeans did not adopt these digits because they relied so heavily on the abacus to carry out calculations. Despite all the hostility toward the new numerals, they were eventually accepted because of their practicality and simplicity compared to previous number systems. [[56]](#footnote-56) They were immediately put to practical use in Italy at the end of the thirteenth century, as traders and merchants used them for everyday calculations.[[57]](#footnote-57) Al-Khwarizmi’s emphasis on the simplicity and the functional application of his numerals, were realized and adopted by Europe and other cultures.

Al-Khwarizmi used mathematics to predict the motion of planetary objects and many cultures adopted his astronomical tables for their purposes. He wrote a composition called the Zij *al-Sindhind*. This composition is the earliest extant original work of Islamic Astronomy. Al-Khwarizmi’s Zij *al-Sindhind* was directly copied and revised by many other astronomers to further their mathematical thinking. Ibn Mu’adh al-Jayyani, a leading Andalusian astronomer of his time, comprised a work called the “Tabulae Jahenen” based directly off al-Khwarizmi’s *Sindhind*. Al-Jayyani furthered the mathematical thinking developed by al-Khwarizmi by adapting the Zij for the longitude of his hometown, Jaen, and for measuring the equinoxes.[[58]](#footnote-58) The Tabulae Jahenen serves as another example of how Khwarizmi’s work was copied and improved to further develop mathematical thinking while also simplifying the lives of the people that utilized his ideas.

The Jewish people were also influenced by the work of al-Khwarizmi for measuring. The oldest mathematical text in Hebrew is called the *Mishnat ha-Middol*. This book, written in the latter half of the eleventh century by Abraham bar Hiyya and Abraham ibn Ezra, gives practical rules for mensuration, the art of measuring. It also contains a section on the solution of quadratic equations with geometric analogues. This section is said to have been a later entry into the book because it “was strongly influenced by al-Khwarizmi’s algebra.”[[59]](#footnote-59) Abraham bar Hiyya also wrote a second text titled *Hibbur ha-Meshiha we ha Tisboret*, “The Composition on Geometrical Measures”, while he was studying in Spain. This text includes an extensive coverage of algebra very similar to that of al-Khwarizmi. The text also covers mensuration on plane figures and solids, including pyramids and trigonometric chords, and a piece on Euclid’s division of figures. The Jewish people, who adopted the work of al-Khwarizmi, demonstrate how quickly his ideas spread and how easily his ideas were assimilated into new mathematical cultures.

Al-Khwarizmi had a significant influence on the development of language. The new notation of algebra came to be known as that of al-Khwarizmi, evolved to algorithm, a procedure for solving a mathematical problem in a finite number of steps that often involves repetition of an operation. His name, in fact, evolved over time to eventually become algorithm:

“al-Khwarizmi – Alchoarismi – algorismi – algorismus – algorisme – algoritmi – algorism – algorithm.”[[60]](#footnote-60)

As mentioned previously, Arabic became the universal language for scientific inquiry during al-Khwarizmi’s lifetime. Therefore, his own name became a significant element within Arabic sciences and future cultures.

Al-Khwarizmi’s mathematics influenced some of the greatest mathematical thinkers of all time, which illustrates the power of his ideas. Two of them were Greek philosopher and mathematician, Descartes, and one of the greatest mathematicians of all time, Fibonacci. Descartes composed a work called *Rules for the Direction of the Mind.* The section on the *Discourse on Method* includes an appendix titled Geometry. In this section Descartes proved a geometric basis for solving algebraic equations, which to a great extent had already been done by his predecessors as far back as al-Khwarizmi.[[61]](#footnote-61) Descartes ideas of geometry were crucial to his philosophical ideas so it was important that his ideas and calculations were easy to understand and apply for practical purposes.

Fibonacci was influenced so greatly by the Hindu-Arabic numerals of al-Khwarizmi that he strongly encouraged their usage. Fibonacci wrote *The Book on Calculation*, which was the first work in Christian Europe on algebra and geometry. The fifteenth and final chapter of Fibonacci’s influential book, *Liber Abbaci*, depended heavily on the geometry and algebra previously established by al-Khwarizmi. Fibonacci also spread the Hindu-Arabic numerals that al-Khwarizmi initially wrote about, with his statement where he says,

“The nine Indian numerals are 9,8,7,6,5,4,3,2,1. With these nine and 0, any desired number can be written.”[[62]](#footnote-62)

With this statement Fibonacci launched the revolution of mathematical numerals with the Indian numerals at the helm. From Fibonacci influence encouraged the transition to new numerals from traditional numerals.

Therefore, al-Khwarizmi revolutionized mathematical thinking because his new algebra and numerals emphasized practical applications that were previously overlooked by other intelligent cultures. The wealth of knowledge in the House of Wisdom, demonstrated Babylonian, Greek, Indian, and Arabic mathematical ability. However al-Khwarizmi adopted the wisdom from his predecessors to create a systematic approach to algebra and to illustrate the more efficient Indian numerals. In developing his new method, he stressed the easiness and simplicity to solve everyday problems, rather than its theoretical function. Consequently, the mathematical ideas of al-Khwarizmi quickly spread, as other mathematicians realized the improvements that algebra offered compared to long-standing tradition of geometry. Al-Khwarizmi’s development of algebra was revised and improved by every culture which initiated the advancement of mathematics to an unprecedented level.

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1. Crest of the Peacock Page 450 [↑](#footnote-ref-1)
2. Crest of the Peacock Page 450 [↑](#footnote-ref-2)
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