How Mathematics and Art

are Interconnected in al-Andalus

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 Antonio Fernandez Puerto, Professor at the University of Grenada, once said, “When you go to a concert, and you listen to Mozart…perhaps you don't know…music, but you notice that there is something magic. The same thing happens here in the Alhambra, you can feel it” (Copestake). The “magic” Fernandez-Puerto is describing represents the subtle mathematics the average person misses when they look at the Alhambra. In fact, the Alhambra was designed completely around mathematical and geometrical concepts that were incorporated into southern Spain’s culture during the Muslim rule. In the early 700s, the Muslims conquered Spain, setting the stage for intellectual development. The scholars in al-Andalus were able to work together in favor of academic advancement, filling southern Spain with an overwhelming air of mathematical reasoning. The mathematical beauty in the Alhambra was able to thrive due mostly to scientific developments of the Muslim, Christian and Jewish scholars in al-Andalus.

 The Muslims are a group of people who follow the word of the prophet Muhammad, and after his death in 623 CE in Mecca, many Muslim leaders came to power wanting to spread Islam and Muslim influence. In 711 CE, Jews and Christians occupied most of Spain, and when the Muslims invaded and took control of Spain, peace within all religions was encouraged. From the perspective of the Muslims, the Christians and Jews were not so different since the Qur’an is based around the same character as in the Torah and the Old Testament – Abraham. Therefore, the Muslims accepted those already living in Spain as “Aha al-Kitab”, meaning “people of the Book” (Lowney 38). The basis for religious tolerance among the three major religions in al-Andalus began from day one.

 In the short length of this paper, it seems only appropriate to discuss in depth only two of the many scholars who lived and thrived during medieval Spain. Therefore, Maslamah Ibn Ahmad al-Majrit (al-Majriti), an astronomer, mathematician and chemist, and Abu Ishaq Ibrahim ibn Yahya al-Zarqali (al-Zarqali), an astronomer, will be the main focuses for this section of the paper.

 Al-Majriti was a famous mathematician, astronomer and chemist, born in about 950 CE in Madrid. Al-Majriti’s major contribution to the intellectual society in al-Andalus was the expansion of the work of the Greek astronomer, Ptolemy. Known formally as Claudius Ptolemaeus, Ptolemy was a brilliant astronomer who showed “for the first time in history…how to convert specific observational data into the numerical parameters for his planetary models, and with the models to construct tables whereby the solar, lunar, and planetary positions and eclipses could be calculated for any given time, past or future.” (Gingerich 1). Ptolemy was well known at the time for his book, *Syntaxis*, commonly referred to as the *Almagest*, and scholarly work on it during the Muslim rule of Spain helped it spread and grow.

 *Almagest* is a scientific text, divided into thirteen books that served as a basis for astronomical and astrological study at the time. Ptolemy’s work was so revolutionary because at the time, he had no predecessors to the topic. *Almagest* first lists a table of sines and discusses “the great circles encountered in the visible heavens- most obviously the horizon and then the equator, the ecliptic and the meridians” (Bulmer-Thomas 300). The rest of the twelve books discuss theories about human perspective to space, sun, moon and stars and various phenomena of planets.

 Al-Majriti’s work on the *Almagest* and was an essential part of al-Andalus’s history. Al-Majriti first translated *Almagest* from Greek into Arabic, a feat that took him many years to accomplish. He greatly criticized Ptolemy’s ideas once he had translated them, especially the fact that Ptolemy had placed the Earth in the center of the universe. An eleventh-century historian, Ibn Sa’id of Toledo, stated al-Majriti’s major contribution occurred by “apply[ing] himself to the observation of the heavenly bodies to understand the book of Ptolemy”, though no mention of his inability to come up with an alternate to Ptolemy’s ideas Is mentioned (Freely 110).

 Later in his life, al-Majriti studied al-Khwarizmi’s astronomical tables and edited them so they were could be studied further by scholars in Cordoba (Freely 110). Al-Khwarizmi was an astronomer, mathematician, geographer, and was considered to be one of the most significant Muslim scientists. He is cited as the inventor, or father, of algebra, and the man to popularize the “Hindu-Arabic” numeral system (Brezina, 11). Al-Majriti studied his work, *Zij al-Sindhind*, which contained “tables for computing eclipses, solar declination and rich ascension, and various trigonometrical tables,” and was expected to have been influenced by Ptolemy. The Indian people regarded this piece highly because it was derived from Hindu astronomy, which played a major role in their religion. (Singh, Kirmani 574) By modifying *Zij al-Sindhind* so it could be available to scholars in Cordoba, Al-Majriti brought Persian mathematics into al-Andalus, a new style that was studied in greater detail later in Toledo, Spain.

 The other main scholarly focus of this paper is al-Zarqali. Al-Zarqali, born in Toledo, in 1028, worked extensively on mathematical and astronomical studies. Al-Zarqali wrote six works on mathematical astronomy and instruments required for astronomical study (Freely 112). He studied Babylonian astronomy, and elaborated on Ammonius’s *Almanac*, a work that contained “tables of sines, cosines, versed sines, secants, and tangents” (Singh, Kirmani 1143). His greatest contribution, however, probably was the construction of the *Toledan Tables.* Through strict observation and exact mathematical computations, al-Zarqali created a table that allowed for a multitude of aspects about many planets to be determined at any given time. The *Toledan Tables* are an compendium of astronomical information: they contain calendar conversions, daily movements of planets, eclipses, and specifics about various planets – such as altitude, visibility and velocity at any given time (O’Callaghan1247).

 Al-Zarqali’s *Toledan Tables* superseded al-Majriti and al-Khwarizmi’s *Sindhind*. Though containing borrowed information from *Sindhind* and *Almagest*, the *Toledan Tables* make their predecessors’ works seem incomplete. It has a table of sine functions based of the work of al-Khwarizmi, which served as a basis to determine how far a planet is. Used in conjunction with basic ideas from Ptolemy, such as spherical geometry, the *Toledan Tables* tie together the work of astronomers who lived hundreds of years before to relevant studies of the time.Through the work of his *Toledan Tables*, Al-Zarqali combined the work of Greek, Persian and Spanish-Muslim scholars to increase mathematical knowledge in al-Andalus (Benson, Constable, Lanham 479).

 The mathematic and science-based studies that occurred in al-Andalus influenced developments in other fields. “As Islamic science and learning – as manifested in astrolabes, astronomical tables, translations of Aristotle, mathematics, and technology for the manufacture of paper – passed through [Europe]…al-Andalus [became] an exotic, elegant, outpost” for mathematical representation in art (Dodds xxii). This math is observed in four different elementary aspects of art in al-Andalus: vegetal forms, calligraphy, figural designs and geometric patterns (Abrams 87).

 The Alhambra and the Great Mosque of Cordoba are two of the many buildings in southern Spain where the effect of Muslim rule and mathematical development is evident. The Alhambra, built in the mid fourteenth century in the Sierra Nevada in Grenada, was designed as a palace to serve as a small Islamic city. Within the Alhambra, arabesque designs are quite common. Arabesque is a style of art that weaves geometric and floral motifs together. Arabesque is commonly found on boxes and canisters in the Alhambra, and is considered the “most genuine type of Andalusian decoration”, since it combined an Eastern tradition with Cordovan styles (Allet 248). The Eastern tradition comes from the idea that spirals, typically found in an Arabesque, represent the cycle of life: “the embody[ing of] the eddying process of Creation’s expansion and contraction” (Sutton 14). Examples of spiraling, flowery Arabesque patterns are evident all throughout al-Andalus, especially in The Great Mosque of Cordoba.

 According to legend, the Great Mosque of Cordoba was built by Abd al-Rahman I in about 787 CE (Dodds 12). Today it is considered to be the leading Islamic artistic accomplishment in Spain (Altet 203). The mosque was built in such way so two naves intersected the prayer wall, called al-qibla. On this al-qibla, inscriptions made from the Qur’an, allowing the word of the Book to be a part of the “visual dialogue” presented in the mosque (Dodds 22). The use of inscriptions from the Qur’an became a standard practice for Muslim artists, especially as time progressed, though it is still unclear whether the Great Mosque of Cordoba is the founding location of these inscriptions.

 The “figural designs” and “geometric patterns” that Dodds talks about are more relevant to the topic of this paper, since they are a directly influenced by the scholarly work done in al-Andalus. From the early 700s to the end of the 1300s, the Muslims expressed a need for intellectual growth, and scholars like al-Majriti, al-Zarqali, and countless others followed through on that need. The scholars brought together a community of academics who solved mathematical problems in abstract ways – primarily in astronomy. However, artists of the time felt this same need in the realm of art, and they responded by subtly basing their artwork around mathematics.

 The mathematics seen in the art found in the Alhambra and in the Great Mosque of Cordoba used simplified forms of the techniques of the astronomers. Al-Andalusian scholars, who stressed the importance of astronomy and mathematics in life, also had a revolutionary effect on artists – to use circles, perpendiculars, and other geometric figures in their art. Most of their geometric figures come in the form of tile mosaics, or figural designs, all requiring complex geometry skills. In order to design these figures, an artist would start with a base shape, usually a rectangle or circle, and use perpendicular or bisecting lines in order to create a lavish design, known as ruler and compass designs (Abrahm, 87).

 The tile mosaics found throughout the Alhambra and The Great Mosque of Cordoba are more complicated than they seem (Lovric 1). For the tile patterns used in al-Andalus, a geometrical shape or a group of shapes, created with the above method, would be used in a tessellation, meaning that shape would tile a floor or wall indefinitely. However, instead of the copy of the shape being placed right next to the original shape in a linear fashion, the artists could rotate or reflect the image about a point or line, respectively, creating a completely different image. In fact, there is a combination of seventeen rotations, reflections and glides that will create a complete tessellation. That is to say, seventeen completely different patterns can be created, and though it is somewhat debated, all seventeen are believed to exist within the Alhambra (Lovric 3).

 A basic tessellation seen all throughout the tile work in the Alhambra is explained in great detail in Daud Sutton’s Book, *Islam* *Design: a Genius for Geometry*. He describes in depth how ruler and compass designs are created to from more than just basic shapes; they were mostly used to create infinite, complicated tessellations themselves. For example, creating and repeating a six-pointed star surrounded by six hexagons can be done using a very simple tessellation. Though the tessellation looks complex, the artist followed a very simple pattern, starting with a circle that intersects another identical circle, so their centers are exactly one radius apart. At the intersecting point of the two circles, the center of another circle is assigned, and the compass is used to draw another identical circle. In other words, the intersection of two circles represents the center of another circle, allowing the pattern to extend as far as the artist wanted it to (Sutton 2). Although it is easy to comprehend the geometry behind this pattern of intersecting circles, depending where the artist draws lines through these circles determines the internal shapes. For example, using the method just discussed, drawing lines that intersect the circles forming hexagons makes the aforementioned pattern. However, if lines were drawn to make rectangles, or triangles, or any possible other shape, a completely different pattern could emerge, which in turn has the potential to form the seventeen mosaic patterns (Sutton 4).

Mangho Ahuja and A. L. Loeb, co-authors of *Tessellations in Islamic Calligraphy,* once said, “tessellations are just one such example of the application of geometry to create beautiful art. A… tessellation is a rarity, because it requires expertise in…geometry and calligraphy.” Their quote can be directly applied to the tessellations done through tile work and geometric patters in the Alhambra and the Great Mosque of Cordoba. Though beautiful and artistic, the creation of perfect geometric shapes involves complex mathematical properties such as distance, arc measure, volume, area, angles and various others, makes Muslim tessellations in al-Andalus forever inspiring.

 Without the Muslim occupation of Spain, the math-based art in al-Andalus would most likely never have surfaced. One can be certain that the Muslim rule in Spain encouraged this style of art to flourish. The scholars, al-Majriti and al-Zarqali, among many others, worked their entire lives expanding on and writing new scientific works, effectively transforming al-Andalus into a pivotal center for mathematical learning and development. Naturally, aspects from the scientific fields, such as arc lengths and the sine function, translated into art, opening a new pathway for artists in al-Andalus. In order to fully appreciate the beauty of the Alhambra or The Great Mosque of Cordoba, one must understand the relationship between the scholars and the artists that thrived under the Muslim rule of Spain.

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