Peter Zona

November 19, 2010

Mathematics Over Time

Professor Little

The Importance of Geometry

Since the time in ancient Alexandria when Euclid first produced his great work, the *Elements*, many students have asked why studying math like this is necessary. More specifically, students have questioned the importance of geometry and geometric proofs. The *Elements* is a standard textbook of elementary mathematics. Over the centuries that have passed since Euclid produced his ideas, countless mathematicians have done their best to put into words the beauty of geometry and why it is as important as it is. A few of these were the Latin author Stobaeus, the Greek philosopher and mathematician Proclus, the poet Edna St. Vincent Millay, and the French mathematician, author, and philosopher Blaise Pascal. While Euclid’s *Elements* is best known for explaining geometry in the form of definitions, common notions, postulates, propositions, and proofs, there is a lot more to it than that. The work includes plenty of algebra and even discusses number theory in the sense of greatest common denominator, prime numbers, and factorization. He also touches on irrational numbers. His work has been so influential over the years because it is the closest thing we have to knowledge that is completely certain. Nevertheless geometry is the main focus of the *Elements* and brings to question why it is necessary and what purpose geometry serves in the world.

In the fifth century C.E. the Latin author Stobaeus recorded a famous story that had been passed down for several generations. He wrote, “Someone who had begun to read geometry with Euclid, when he had learnt the first theorem, asked Euclid, ‘what shall I get by learning these things?’ Euclid called his slave and said, ‘Give this man three obols, since he must profit from what he learns.’” This story is discussing a person who began to study geometry with Euclid. Upon learning Euclid’s first theorem, he inquired as to what he would receive by learning the propositions. Euclid responded by instructing his slave to give the man three obols, about the same as three pennies. This response shows that knowledge is a greater profit than money. He tried to display this to the student by harshly demonstrating that people will not be paid in monetary terms for learning the theorems but instead be rewarded with knowledge. Euclid would respond to the question regarding the reasons behind studying geometry by saying that people should study geometry to know the basic mathematical truths and realize why they are true before putting this knowledge to use in the real world. As the target of this answer, a suitable response would be that geometry serves no purpose in the real world and is simply a waste of time. As the target of this means of reply, one could feel that Euclid is insulting him rather than giving a straight answer. However, at the same time, Euclid could feel that the student is insulting him and his work by asking such a question. While I have often wondered and questioned the reasons behind studying geometry, I have only recently come to realize its necessity. In addition to being able to prove mathematical statements to be true, geometry also teaches a way of learning how to think logically and express a series of steps to explain why something is the way it is.

Also in the fifth century C.E. the Greek philosopher and mathematician Proclus expressed his thoughts on Euclid and the *Elements*. He wrote, “Mathematical science must be considered desirable in itself, though not with reference to the needs of daily life. If it is necessary to refer the benefit arising from it to something else, we must connect that benefit with intellectual knowledge, to which it leads the way and is a propaedeutic, clearing the eye of the soul and taking away the impediments which the senses place in the way of the knowledge of universals.” He is saying that mathematical science is something that should be desired on its own. It is not something that should be learned strictly because of the needs of everyday life. The great benefit of mathematics is that it provides people with intellectual knowledge. It is also something that is studied to prepare to learn something else. It takes away sensual experiences and allows the mind to uncover the truths of the universe. During our trip through “Book One” of the *Elements*, the “eye of my soul” has been cleared in a sense. I have come to realize why we must understand the core geometric truths and their impact on everyday life. The senses can place impediments in the way of true knowledge by making people see things a certain way that is in fact incorrect. With the “knowledge of universals,” Proclus is referring to basic truths about the world that can only be uncovered through a logical method and not by our senses. Geometric proofs are an example of some basic truths that can be difficult to uncover, but once recognized, make perfect sense and are easy to follow towards uncovering a mathematical truth. I think that this is a sufficient enough reason to study geometry. It is a process that forces logical thinking in a series of steps to reach a true conclusion that is always correct, no matter the situation. It allows a person to reach an unbiased truth about something, unaffected by the senses.

During the 1600’s, a French mathematician, author, and philosopher named Blaise Pascal revealed his thoughts about mathematics. He said, “Mathematicians, who are only mathematicians, have exact minds, provided all things are explained to them by means of definitions and axioms; otherwise they are quite inaccurate and insufferable, for they are only right when the principles are quite clear.” He is saying that those people that are strictly mathematicians can only come up with solutions when all of the facts are put in front of them. They are incapable of figuring anything out otherwise. This goes against the thoughts of Stobaeus and Proclus. They felt that our reason for studying geometry, specifically the process of finding proofs of statements, improves our process of logical reasoning. This process can then be put to use in other aspects of life. With the “definitions and axioms,” Pascal is getting at the fact that known, confirmed statements are necessary for a mathematician to begin to discuss something. Geometric proofs may not be the best training in the sort of thinking that is necessary for solving hard real world questions. It restricts the process to requiring that some of the statements are already true and impossible to prove wrong. It is possible that the subject of philosophy could be better than geometry at preparing students to answer these kinds of questions. Nevertheless, most educators agree that geometry is better because all students study it in high school.

In sum, geometry is a subject that is of great importance in everyday life. While it may at times seem to be unnecessary, over the centuries it has been used in many different ways to uncover the answers to several of the Earth’s mysteries. During the Hellenistic Period, Aristarchus used geometry to determine that the sun is the actual center of the Solar System instead of the common notion at the time that everything revolved around the Earth. In addition, Eratosthenes proved that the Earth is spherical as opposed to flat, another notion that many people believed at the time. It was geometry and Euclid’s work that allowed these discoveries to be made in addition to those by Heron and Ptolemy. While no further major discoveries were made until the Renaissance period of the fourteenth and fifteenth centuries, geometry was still prevalent in society and remains so until today. Geometry allows for architecture and art to be created in a more efficient way that is both stronger and more appealing. It also facilitates progress in these areas. Additionally, as Stobaeus and Proclus suggested in their writings, geometry serves the purpose of teaching a process of logical thinking from a given starting point to reach a conclusion. While Blaise Pascal suggests that philosophy may be a better subject for teaching these skills, this is not necessarily the case. Philosophy does teach a progress of logical thought, but the statements can always be argued as philosophical propositions cannot be proven to be true or false. With geometry, the final outcome of the proof is always certain. While some aspects of geometry may seem to be unnecessary and useless, it is nevertheless knowledge that all people must obtain to understand truths about the world.