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Math Across Time and Cultures

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Why Study Euclidian Geometry?

Euclid’s *Elements* is one of the most well known and studied works in history, arguably second only to the Bible. For centuries, students of the sciences throughout the world used his book exclusively as a study guide for geometry. But why was the *Elements* so influential back then, and how has it become the foundation for countless other mathematical and scientific texts and even disciplines? Perhaps it is because Euclid had the ability to take a large realm of concepts and break it down into its most basic, defined principles and axioms. He creates a beautiful masterpiece of a mathematical world in which everything is justified or supported by unquestionable, logical fact, something appealing to people living in such an imperfect reality.

In the beginning of the *Elements*, Euclid presents the basic postulates, common notions, and definitions that will later provide support for his geometric propositions. For students, this brings an element of clarity and organization to the study of geometry; without them the more complex ideas Euclid proposes would be unjustified. This method of starting with the most basic “building blocks” in any subject or task in order to reach higher levels of understanding even when dealing with more complex ideas is often used in the argument that mathematical thought is the basis for obtaining all knowledge. Blaise Pascal disagrees, however, stating that people who think like mathematicians “are quite inaccurate and insufferable, for they are only right when the principles are quite clear.” In reality, the “principles” are not always laid out for us before we attempt to move on to greater complexities; if we were to think like mathematicians in these circumstances, we would not be able to solve the problems facing us. Not every challenge we encounter can be treated like a geometric proof, with each step clear-cut and rationally thought out. However, when it comes to learning, this can be the best way to work. The basics, the axioms of a subject, ought to be learned before a student begins to apply those principles to larger issues. Just as Euclid gives his students definitions and postulates in *Elements* before moving on to the propositions, children today are given the alphabet and tools for writing before they are asked to compose an essay. In this manner, we can organize what we come to know.

It is important that what we know is truth, or at least that it is as close to truth as we can come. Although scientists in the nineteenth century began to discover that there were other forms of geometry which were more applicable than Euclid’s in the natural world, the *Elements* still allows its students to study absolute geometric truths. These truth, the definitions, common notions, and postulates, are given in an order that is crucial to his method of validating the propositions he makes. He starts with definitions of the point, line, and different geometric figures. These, like any definition we are given, can be subjected to rational questioning if one wishes to be fastidiously critical. However, it does not seem difficult or risky to accept them as truths, because they are simple, straightforward, and in agreement with what our brains tell us “makes sense.” Similarly, the common notions or “axioms” presented are very self-evident and do not leave much room for skepticism, and the postulates are called such because they are to be assumed true. As a result, when working toward an understanding of each proposition, we can assume that each step of the proof is true because it is made up of other truths that we have already accepted.

Possession of more or less unquestionable knowledge, knowledge of truth, is a wonderful thing. If we know about geometric truths, they can be applied to other sciences to make assumptions about the physical universe. During the Hellenistic Period, many advancements were made by Greek thinkers using Euclid’s *Elements* which we still make use of in modern times. Aristarchus used geometry to conclude that the sun was the center of our solar system, and Eratosthenes estimated the circumference of our spherical Earth based on knowledge acquired from the *Elements.* Ptolemy explained the seemingly retrograde motion of the planets with it, and Archimedes and Huron’s machines sprung from it. Without these discoveries we would be lost, but it is important to appreciate Euclid’s work for intellectual as well as practical purposes. Proclus, a philosopher and mathematician living in ancient Greece, stated:

“Mathematical science must be considered desirable in itself, though not with reference to the needs of daily life. If it is necessary to refer the benefit arising from it to something else, we must connect that benefit with intellectual knowledge, to which it leads the way and is a propaedeutic, clearing the eye of the soul and taking away the impediments which the senses place in the way of the knowledge of universals.”

The study of Euclidean geometry should not purely be for the purpose of understanding mathematical concepts and applying them to other sciences. It demonstrates that the human mind is capable of logic that, when used properly and in an organized fashion, can ultimately bring us to the “knowledge of universals”; not only geometric universals but also universal truths. A mind practiced in the art of induction and deduction can eventually find answers to greater questions about human nature and purpose in our existence.

Euclid’s *Elements* is not the only “propaedeutic” that exists in this world, but it may be the most beautiful. Edna St. Vincent Millay articulated the fact that not a single person besides Euclid has ever seen such raw, unpolluted beauty in her poetry: “Euclid alone has looked on Beauty bare.” The book is one of both mathematical elegance and elegance in the real sense of the word. Like a pair of ballroom dancers, the *Elements* fuels off of each step and flows easily from one to the next. But it also proves a multitude of geometric propositions using an almost unbelievably small number of assumptions. Each proof’s beauty exists in its simplicity, and Euclid’s genius shines through in his ability to take pure, “bare” geometric ideas and compile them to demonstrate something more complex.

After studying Euclid’s *Elements* for weeks on end, a student can lose sight of its benefits if they get caught up on each step of each proof. Geometry and its practical applications have their place in the natural world, as the great thinkers of the past have proven to us. But as students of Euclid, we must instead try to focus on how each level of logic flows into the next to create a cohesive idea, and how the smallest truths, when compiled, can lead to an understanding of greater, universal truth.