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Platonic Solids

In the famous painting **The School of Athens** by the Italian Renaissance artist Raphael, Plato and his student Aristotle are placed at the center of the vanishing point with their hands gesturing at different aspects of their philosophy. Plato with his finger pointing upward to the vault above suggests that there is an eternal world different from the physical world. The eternal world, he believed, is one that never changes and is seen as the “real” world of forms where true knowledge can be obtained. The physical one is the opposite and is considered as only a mere copy or abstraction of what is real. Aristotle, in contrast, has a more empirical view, gesturing to the physical world as the real world where true knowledge can be deprived through experiences and observations.

These distinct ideas, depicted through the two characters in the painting, can also be applied to the mathematical and scientific world. Mathematics through its abstract proofs and formulas has been said to be the closest to absolute truth that we can ever obtain about the natural world. Therefore, it can be represented as the eternal world. Science on the other hand is governed mostly by observations and theories about the physical world through experiences and therefore resembles Aristotle’s idea.

Although these distinct views seem to be in conflict with one another, in the mathematical and scientific sense they combine to form theories about what many of us today perceive as truths.

The platonic solids in particular have a combination of both worlds mixed together. The platonic solids, named after Plato, are geometric figures with congruent sides, angles, and edges. As was proven in Euclid’s **Elements** volume **XIII**, there can only be five of them. Since the angles at the vertices of all the faces have to be congruent, each angle has to be less than 120 degrees. Adding more sides would cause the three dimensional shape to add up to angles greater that 360 degrees or four right angles. This cannot occur since a sphere is already 360 degrees and each of the figures in the proofs is inscribed in one. Having shapes different from the five platonic solid would also cause it to not close properly and therefore it would not fit the definition of the platonic solid.

A later proof came from Leonhard Euler. He found the relationship between the number of vertices, edges, and faces of the platonic solids with a simple connection that the vertices minus the edges plus the number of faces would equal two for every one of figures. Using this simple connection, which had been found earlier by Rene Decartes (Aczel page 229), we can prove that the figures can only form five different shapes using the formula, 1/m + 1/n=1/2 +1/E. This fact has not changed or been modified.

The platonic solids have been believed to have held the secrets of the universe. The Greeks in particular believed that the structure of the universe was made up of the five elements. To Plato himself, they represented the four elements fire, water, air and earth. The tetrahedron was associated with fire, the icosahedrons with water, the octahedron with air, and the cube was compared with the earth. The final Platonic solid, the dodecahedron, was said to represent the universe as a whole or the heavens.

This belief seems to gesture to an eternal world of forms that is supposed to represent truth. Yet, when looking at the periodic table today, we can see that the theory is false. Over centuries, scientists have tried desperately to apply the four elements of fire, air, earth, and water to structure of the universe. But, their attempts have often led them astray. As we see today, there are about 112 known elements. These combined are what make up the macroscopic world we live in today.

Another theory that related the platonic solids to the mathematical structure of the universe came from Johannes Kepler who hypothesized that there was a connection between the mystical shapes and the orbits of the planets (Aczel page 222). From the study of Euclid’s elements, Kepler was aware that the platonic shapes could be inscribed perfectly in a sphere. Each figure could be inscribed in a larger sphere, which when combined would create a sequence of nested spheres.

He was motivated to thread together the patterns in the celestial planets, so he came up with a model of the universe. The five platonic solids, he suggested, could fit inside the sphere of the planets Mercury, Venus, Mars, Saturn, Jupiter, and the Earth. With the sun at the center, Kepler visualized, using the five unique shapes, a mathematical (eternal) world that cannot be easily observed in the physical world. He published his theory in 1596 in what is known as the **Mysterium cosmographicum** (Aczel page 223). Later improvements in the second edition of the book, however, showed the inaccuracies in his initial depiction of the universe.

Despite the inaccuracies in platonic solid theories established by earlier scientists about the universe, these five figures do appear in nature. One perfectly good example would be the different structure of crystals. The anions, depending on the size of the atoms, form different types of holes such as the tetrahedral or octahedral holes that the cation can fit in. Other examples appear in the molecular bonding of the atoms and many viruses also have similar shape to the platonic solids. The herpes virus for example has the shape of the regular icosahedrons.

Altogether, the knowledge obtained from the study of the figures seems to be pointing to two worlds, one eternal that comes from the human mind and the physical world perceived through observations. The eternal world of forms can be seen through the proofs given, especially in Euclid’s Elements XIII, which has lasted for centuries. The depiction that these figures are perfectly symmetrical and congruent has also remained unchanged. Another aspect of our knowledge comes from observations. But these sometimes lead to inaccuracies, which has to be modified later on. So, which knowledge represents truth?

The issue raised by the two philosophies leads to a question on of the nature of reality. Mainly, what is true knowledge and where is it deprived from? As was stated by Plato, “The object of knowledge is what exists and its function to know about reality.” Yet, sometimes reality is distorted by the human mind. So, what is truth and reality?

In the famous painting **The School of Athens** by the Italian Renaissance artist Raphael, of all the great and talented thinkers of the ancient Greek, Plato and his student Aristotle are placed directly at the center with their hands gesturing at different aspects of their philosophies. To Plato, true knowledge exists in the eternal world, which is where truths can be obtained through reasoning. The physical world, he believed, is only a mere copy of it. Aristotle, on the other hand, points to the physical world as the place where true knowledge can be deprived through experiences and observations.

Both philosophies points to different aspects of how knowledge that can be obtained in a lot of great minds. One of it can be found in the eternal world, governed by reasoning, which can be thought of as within the human mind. Another is through observations and experiences from the physical world. Both of these combined lead to knowledge that many of us today see as reality.

Sources

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