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Math Across Time

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Consequences of Non-Euclidian Geometry

In textbooks such as Elements, Euclid created a foundation of geometric knowledge based on axioms and definitions to prove more complex geometric knowledge. With the perspective of geometric certainty, supporters believed Euclid had created knowledge that described absolute knowledge about physical world ( ). Rationalists, such as Plato, adopted geometry to prove that accurate knowledge is obtained with reason and logic. In contrast, empiricists argued for the necessity of senses to gain knowledge through repetitive experiences. Regardless the empirical argument, geometry defended rationalism as the unarguable evidence that geometry accurately portrayed the physical world. However, the knowledge of non-Euclidean geometry contradicted Euclidean geometry as an accurate deduction about the physical world. Therefore discovery of non-Euclidean geometry, consequently disproved Euclidean geometry, and proved that man obtained knowledge through his senses and not through reason.

Plato’s Meno provides an argument for rationalism through the recollection of universal forms, such as math and geometry. In the Meno, Plato’s character Socrates proves that a slave boy who has never had the experience of learning math in school still knows the area of the squares ( ). Using this example, Plato illustrates that the slave boy has knowledge without directly experiencing it through education. Plato claims that the slave boy knows the areas through the recollection of everlasting true knowledge, or what he calls the universal form. Plato uses the idea of recollection of the universal forms to justify the concept of rationalism to apprehend the forms and to gain knowledge. Plato also claims that knowledge of geometry, just like math, is a universal form because geometry provides accurate knowledge about the physical world ( ). By referencing math and geometry, Plato is able to create an argument for man’s ability to gain knowledge of the universal forms with rationalism. Plato believes that man can only have access to knowledge through reason and logical deductions. However with the discovery of non-Euclidean, Plato’s argument for rationalism has been disproven. Non-Euclidean geometry proves that Euclidean geometry does not provide knowledge about the physical world, since non-Euclidean geometry is also mathematically accurate.

However before the knowledge of non-Euclidean geometry, Euclidean geometry was a critical asset for the argument of rationalism. Rationalists believe through the assistance of reason, man is able to gain knowledge of absolute truths without relying sense experiences (326). Euclidean geometry justified rationalism because rationalists relied on geometry to provide knowledge of the physical world. Rationalists such as Descartes claimed that man is unable to trust his senses because his senses can be deceiving. Descartes creates the Dream Argument to illustrate that man cannot trust his sense to obtain knowledge because his senses are unable to distinguish the difference between when he is awake and not sleeping ( ). The deceptiveness of senses can be seen when Spinoza writes, “I may be mistaken in thinking that I am sitting at my writing desk composing this sentence, and I surely may be mistaken that the sun will rise tomorrow, but by no means can I be mistake in my knowledge that the angle sum in a triangle equals a straight angle” (326). Rationalists portray senses as inconsistent and unreliable to represent the uncertainty of knowledge gained through senses and experiences. Therefore without the validity of senses, rationalists can only use their intellect and reason to gain knowledge. Like Spinoza, rationalists justify that the knowledge of geometry is true regardless of the skepticism in his senses. The unarguable evidence that Euclidean geometry was certain knowledge proved that geometry was an accurate deduction of physical world. However, with the advent of non-Euclidean geometry, man cannot be certain that the interior angles of a triangle and the angle of a straight line are equal to the sum of two right angles. Non-Euclidean geometry contradicts Spinoza’s certain knowledge of geometrical angle measurements by providing counter examples. Since the argument for Euclidean geometry no longer applies, rationalists can no longer assert that Euclidean geometry is an accurate portrayal of the physical world.

Before the discovery of non-Euclidean math, empiricists could never completely disprove the argument that Euclidean geometry was a valid deduction about the physical world. Unlike, rationalists, empiricists argued that all knowledge, except mathematical knowledge, comes from sense experiences (327). Supporters for empiricism believed that through repetitive tested experiences with man’s senses, experiments, man can make conclusions about the physical world. However, geometry had always been a contradiction for empiricists like Hume, because of the long-standing perception that geometrical knowledge could be obtained independent of the senses. With the discovery of non-Euclidean, empiricists could disprove rationalism and Euclidean because geometrical knowledge is no longer an absolute truth about the physical world. Therefore man acquires knowledge by referring to his senses and observations to make conclusions and deductions about the physical world. This use of empiricism can be seen in scientific experiments that require repetitive experiences to gather information, analyze data, and make conclusions ( ). Rationalists no longer have a sound argument for the basis of reason to acquire knowledge about the physical world.

The evidence for non-Euclidean Geometry and for empiricism, consequently justifies the skepticism of all knowledge in the Physical world. In Hume’s argument for knowledge obtained through senses, he develops the Copy Principle, which states that ideas are less vivid copies of impressions or sense experiences ( ). Hume claims that all ideas are derived from experiences, thus knowledge is obtained through experiences. For example, a blind person without the ability to see color cannot have the idea in his head, since a blind person has never directly experienced color ( ). As a result from the Copy Principle, man cannot have knowledge of cause and effect because man cannot directly experience the connection between two separate events with him senses. Man understands the concept of cause and effect through repetitive experiences that causes our mind to form conclusions and make assumptions between two separate events. Therefore since man cannot be certain of cause and effect man cannot be certain of anything beyond the present moment. Man lives in completely uncertainty, skepticism of any events in the future beyond the present. Non-Euclidean geometry proves Hume’s empirical Copy Principle, which causes man to live in the physical world with skepticism. By disproving Euclidean geometry, man cannot be absolutely certain about knowledge in the physical world.

Even though empiricism is able to contradict rationalism through evidence of non-Euclidean Geometry, Kant tries to prove that possibility of knowledge without sense experiences. Kant rejects Hume’s assertion that knowledge is gained through observations with the concept of knowledge “a priori;” knowledge that is “timeless and independent of experience” (329). According to Kant, he still believes that man has certain knowledge that is genetically inherited in all humans before he has experiences. Kant describes two types of knowledge “a priori.” “Analytic a piori” knowledge which is man’s natural ability to have preexisting knowledge of logic, and “synthetic a priori” which is man’s natural ability to have preexisting knowledge of time and space. Kant argues that man has natural instincts regarding space and time, based on Euclidean geometry and arithmetic respectively (329). Despite the proof for empiricism, Kant claims all humans still have genetic awareness of Euclidean geometry independent of human’s senses. However, non-Euclidean Geometry still demonstrates that Euclidean geometry is no longer an accurate portrayal or the physical world. Kant’s argument for the natural human intuition for geometry is invalid because non-Euclidean geometry proves Euclidean geometry to be inaccurate. Humans cannot have pre-existing knowledge of Euclidean geometry as a portrayal of the physical world, because the physical world in not based on Euclidean geometry.

Evidence of non-Euclidean geometry, concludes that empiricism and skepticism is true, yet mathematic and geometric knowledge continues to be perceived as absolute knowledge. From Descartes’ Dream argument, he proves that man’s senses are deceptive and cannot be trusted. Hume expands of this concept to conclude humans must be skeptical of knowledge because absolutely certainty through man’s senses is impossible. Although man cannot be certain of anything according to Hume, man can in fact gather repetitive experiences and make assumptions that are nearly accurate. This slim margin of skepticism causes mathematics to continue without diminishing the value of their math. For example, non-Euclidean Euclidean is only applicable at extremes, such as high velocities or over large distance; mathematicians can still conduct research with relative certainty( ). Math and Euclidean geometry can still be perceived as certain because these rare extreme factors do not create significant errors in their calculations consequently their knowledge.

The knowledge of non-Euclidean geometry discredits Euclidean geometry as knowledge of the physical world. As a rationalist, Plato argues that man gained knowledge of universal truths through reason. Conversely, Empiricists claims that all knowledge derived through sense experiences. Geometry had always been rationalism’s strongest argument and empiricism’s unanswerable counter example until the discovery of non-Euclidean geometry. New non-Euclidean Geometries successfully explained that Euclidean geometry was not an accurate description about the physical world, but it also justified that absolute knowledge is impossible because of deceptive senses. Mathematicians still trust in the relative certainty of math and Euclidean geometry because of repetitive experiences. Even through non-Euclidean geometry ability to have definite knowledge, both math and geometry are perceived to be accurate.