MONT 105N – Analyzing Environmental Data
Individual Problem Set 1 – **due**: Friday, February 1

Directions: Create a text document (preferably using Google Docs) including the plots requested in question B and your answers to the following questions.

A. A common *rule of thumb* is that *the time it takes for an exponentially growing stock to double in size*, the so-called *doubling time*, equals approximately 70 divided by the growth rate per unit time (expressed as a percentage).

(1) (Re-)do Exercise 7 in Chapter 5 of our text *Elementary Mathematical Modeling and Data Analysis with Environmental Applications*.
(2) Then check the accuracy of the *rule of thumb* above if $Q(t)$ is growing at 5% per unit time and 10% per unit time.
(3) Why does the *rule of thumb* work as well as it does?

B. Download a personal copy of the shared Google spreadsheet `Fish Stock Management` from the course homepage. (Please don't edit the shared one!) This incorporates the difference equation model for the fish stocking situation we discussed in class.

- Column C gives number of fish in the lake as a function of time in weeks.
- Column D gives the number of fish removed by fishermen, with the 10% increase from 50 fish to 55 fish starting in Week 6.
- Column G gives the "perceived need for restocking" – the number of fish taken, *averaged over the 5 most recent weeks.*
- Column H gives the Desired Number of fish (always 12·Perceived Need)
- Column I (the "Discrepancy") is the difference between the Desired Number of fish and the actual number of fish present in the lake from Column C.
- Column E gives the orders made by the lake manager to a commercial fish hatchery for fish to restock the lake. The amount of each order is computed by adding the perceived need and a *fraction of the Discrepancy.* The spreadsheet is set up so the fraction is 1/(the number in cell `J2`). I called this the *make-up period.* This make-up period is set to 3 weeks as we discussed in class, but you can change the value in that cell to experiment and answer questions (4) and (5) below. We can think of this as a *response delay* where the orders are planned to make up the discrepancy, but not all at once. We are trying to make up the discrepancy over a period of some number of weeks.
- Column F gives the deliveries from the hatchery. Notice that there is a *5-week delivery delay* between the time the order is sent in and the time the order is delivered.

Using this information, and modifying the spreadsheet as needed, Answer these questions:

(1) Describe in words what is happening to the number of fish on hand in the lake over the 100 weeks shown. Is this necessarily a problem? Why or why not?
(2) Why is the Discrepancy value negative in some weeks? What does that mean in real-world terms?
(3) Suppose the perceived need for restocking is averaged over the most recent *three weeks* instead of *five weeks.* This will involve changing the entries in Column G starting in

row 9, but everything else remains unchanged. Copy the new plot into your document. Also describe how the number of fish on hand changes (relative to the original plot) the number of fish in the lake changes (relative to the original plot shown in the spreadsheet when you downloaded your personal copy)

(4) Now suppose the perceived need for restocking is again averaged over the most recent 5 weeks, but delivery delay is reduced to 3 weeks instead of 5 weeks. (Note: You'll see how and where the delivery delay is included if you look at the entries in Column F, starting at Row 9.) Again, copy the new plot into your document. Also describe how the number of fish on hand changes (relative to the original plot).

(5) Now suppose the response delay described above is decreased from 3 weeks to 1 week, but nothing else is changed. (This means you will want to set the delivery delay back to 5 days and the perceived need is computed by averaging the most recent 5 weeks again.) Roughly speaking this means that the stocking manager is going to try to making up discrepancies between the desired number of fish and the actual number of fish in just one week rather than spreading it over three weeks. Again, copy the new plot into your document. Also describe how the number of fish on hand changes (relative to the original plot). Is this what you thought should happen?

(6) Describe a strategy for reducing the severity of the oscillations in the fish stocks (making the amplitude smaller, and/or the period longer). Discuss. Is this what you would have guessed intuitively.

Note: You may want to look at the discussion of the car dealer's inventory example in the excerpt from Meadows' *Thinking in Systems* on the course homepage.