College of the Holy Cross, Fall Semester 2018
MONT 104N – Modeling the Environment
Solutions for Midterm Exam, October 26

I. In discussing amounts of water needed for irrigation of farmland, a common unit of volume is the acre-foot. One acre-foot of water is the amount of water necessary to cover a perfectly flat field one acre in area, to a depth of one foot. Using the information below, answer questions A and B.

- 1 yard = 3 feet
- 1 acre = 43560 square feet
- 1 meter $\doteq$ 3.28 feet

A. (10) How many cubic yards (yd^3) of water are there in one acre-foot?

Solution: The best way to do this is probably to express the answer in cubic feet first, then convert to cubic yards: 1 acre-foot = (1 acre) × (1 foot) = 43560 cubic feet. Then 1 cubic yard = 3^3 cubic feet = 27 cubic feet, so 1 cubic foot = $\frac{1}{27}$ cubic yard. Hence:

$$43560 \text{ cubic feet} \times \frac{1}{27}\frac{\text{cubic yard}}{\text{cubic foot}} \doteq 1613.3 \text{ cubic yards.}$$

B. (10) How many cubic meters (m^3) of water are there in one acre-foot?

Solution: To convert from cubic feet to cubic meters, use the last given conversion: 1 cubic meter $\doteq 3.28^3$ cubic feet $\doteq$ 35.29 cubic feet. Hence

$$43560 \text{ cubic feet} \times \frac{1}{35.29}\frac{\text{cubic meter}}{\text{cubic foot}} \doteq 1234.3 \text{ cubic meters.}$$

II. According to the 2014 Advancing Sustainable Materials Management factsheet produced by the U.S. Environmental Protection Agency, the total amounts of various types of materials produced, recycled, and disposed of in landfills in the U.S. were as follows. All figures are in units of millions of tons.

Category	**Produced**	**Recycled**	**Landfilled**
Paper and paperboard	68.61	44.40	19.47
Metals	23.26	7.90	12.82
Plastics	33.25	3.17	25.10
Glass	11.48	2.99	7.04
Others	4.44	1.29	2.58

A. (15) Construct one bar chart showing the *percentages* of the materials produced that are recycled and a second chart showing the percentages of these materials that are disposed of in landfills.

Solution: Computing percents we have

Category	**%Recycled**	**%Landfilled**
Paper and paperboard	64.7%	28.4%
Metals	34.0%	55.1%
Plastics	9.5%	75.5%
Glass	26.0%	61.3%
Others	29.1%	55.9%

Note: these don't sum to 100% since some of the materials remain in use or are disposed of by incineration and other methods. The bar chart would have bars of heights showing these percentages in the Recycled column.

B. (5) Which material is landfilled most, proportionally (that is, considering the landfilled amount as a fraction of the amount produced)? Explain.

Solution: The material having the largest percentage of the amount produced going into landfills is the Plastics category, since the 75.5% for Plastics is the largest number in the Landfilled column in the last table.

C. (10) What percentage of the total weight of these classes of materials is recycled? What percentage of the total is landfilled?

Solution: The total amount produced is $68.61 + 23.26 + 33.25 + 11.48 + 4.44 = 141.04$ million tons. The total recycled is $44.4 + 7.9 + 3.17 + 2.99 + 1.29 = 59.75$ million tons. This is $59.75/141.04 \times 100\% \doteq 42.4\%$ of the total produced. Similarly, the total amount landfilled is $19.47 + 12.82 + 25.1 + 7.04 + 2.58 = 67.01$ million tons. This is $67.01/141.04 \times 100\% \doteq 47.5\%$ of the total. Note that you *cannot* do this by averaging the percents computed in part A. The amounts are different so they contribute to the total percent differently.

III.

A. (11) In 2010, 2.74×10^5 hybrid and electric vehicles were sold in the US. In 2015, the number was 4.98×10^5. Construct a linear model for the number of hybrid vehicles sold as a function of t = years after 2010 using this information. What does your model predict about the number of hybrid and electric vehicles sold in 2018?

Solution: The slope is

$$m = \frac{498000 - 274000}{2015 - 2010} = 44800.$$

Using the point-slope form, the linear equation is

$$\text{Hybrids sold} - 274000 = 44800 \cdot (\text{years after 2010}) = 44800t,$$

or

$$\text{Hybrids sold} = 44800t + 274000.$$

Since $2018 - 2010 = 8$. The model would predict hybrid sales of

$$44800 \cdot 8 + 274000 = 632400$$

vehicles this year.

B. The following spreadsheet shows more complete data for the total number of hybrid (and electric) vehicles sold for each year from 2010 through 2015.

[Spreadsheet omitted here.]

Answer each of the following questions based on this information:

1. (3) Based on the scatter plot, the R^2 ("coefficient of determination") is probably:
   between 0 and .25 ____________
   between .25 and .5 ____________
   between .5 and .75 X
   between .75 and 1 ____________
   Note that the least squares trendline (regression line) is not very close to the data points in this case, so R^2 would be significantly less than 1. On the other hand, there is a definite upward trend, so the R^2 value would not be really small either. This is the best estimate. (Note: I also gave 2/3 points for either of the adjacent answers, though.) Using GoogleSheets, the computed value of R^2 is $R^2 \doteq .64$.
2. (3) The least squares regression line for this data set would have a slope m about:
   $m = 60000$ X
   $m = -60000$ ____________
   $m = 30000$ ____________
   The slope is definitely positive, so the second value is ruled out. Looking at where the regression line crosses Year = 2010 and Year = 2011, we can see that the vertical axis value is increasing by about 60000 for that unit change in the year.
3. (3) What are the units of the slope? *Solution:* vehicles per year.

Essay (30)

Why is there great interest in creating synthetic bacteria and other single-cell organisms? For what jobs might they be useful? On the other hand, what potential dangers have people suggested in this kind of work? Considering *Frankenstein* as a warning, is this something we should be pursuing with our technology? Are the potential good effects enough to outweigh the possible dangers?

Model response: Apart from general scientific curiosity and the desire to put our current knowledge of genetics to work by designing a new functioning genome "from the ground up," there is interest in creating synthetic bacteria and other single-cell organisms for their potential uses for things like:

- pollution mitigation (i.e. using the new organisms to capture and dispose of polutants in water)
- production of synthetic pharmaceuticals
- production of synthetic biofuels
- production of polymers and other materials.

All of this would fall under the general category of biological engineering, and there are great potential benefits if such organisms could be safely produced.

On the other hand, the fact that the scientists who created this new bacterial life form do not understand what about 30% of its genetic information does opens up all sorts of possibilities for unintended consequences. It isn't possible to predict what the organism might do in new situations outside the ones where it has been tested, for instance. Might it grow uncontrollably? Might it interact in unforeseen ways with other naturally occurring life forms? Might it mutate into something that produces diseases in humans or other current life forms?

How you come down on whether this sort of experimentation is justified is up to you. There's no one "right answer" to whether we should be pursuing this kind of research. I'll be looking for whether you can articulate a position clearly based on the evidence you have.