College of the Holy Cross, Fall 2016
MONT 107Q – Thinking About Mathematics
Midterm Exam, March 31, 2017

Your Name: _____

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra blank sheet of paper at the end that you can use either as scratch paper or as extra space for your essay. You may detach that if you like, but please put your name on it and hand it in with your exam if you do detach it. The numbers in parentheses next to the questions are there point values (100 points total).

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 20
III	/ 20
IV	/ 40
Total	/100

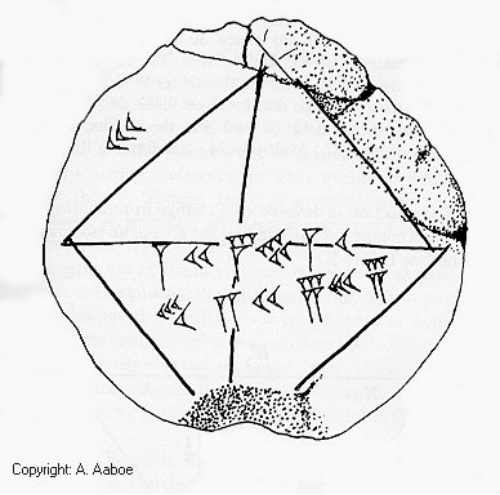

Figure 1: Drawing of the YBC 7289 tablet by Asger Aaboe. I do not have his permission to use this, but he died in 2007, so I think the copyright issue is effectively moot(!)

I. On the Old Babylonian tablet YBC 7289 in Fig 1. above

A) (10) The base-60 form of the number written directly over the diagonal of the square has just one base-60 digit to the left of the semicolon separating the whole number part from the fractional part. Give all the base-60 digits, separated by the semicolon and commas.

B) (10) Express the whole number from part A in base 10. (It's OK to leave this as a sum of fractions if you don't have a calculator.)

II.

A) (5) What does it mean to say that $\sqrt{2}$ and 1 are *incommensurable*?

B) (15) Give the proof the ancient Greeks found for the fact in part A. What consequences did this have for the way they thought of (pure) mathematics?

III. Short answer. Answer any four of the following. If you answer more than four, the best four will be used.

A) (5) What is the main feature that makes the presentation of the number theoretic problems in Diophantus' *Arithmetica* different from earlier mathematics and a forerunner of what we do now.

B) (5) What is the "algebraic form" of the fact proved in Proposition II.4 in Euclid's *Elements*? (The statement: If a straight line is cut at random then the square on the whole is equal to the squares on the pieces and twice the rectangle contained by the pieces.)

C) (5) What are the approximate dates of the YBC 6967 and YBC 7289 tablets? About when did Euclid live?

D) (5) What does the $M^o 20\varsigma 1$ in Diophantus' Proposition I.4 mean?

E) (5) Explain what Al-Khwarizmi gave as a demonstration for his "squares plus roots equals numbers" case of solving quadratic equations. (Recall that he actually did this in two different ways. Either one is OK as an answer here; it is not necessary to discuss both.)

F) (5) How would the Old Babylonian scribes have approached a large calculation such as multiplying $23, 41; 16$ by $17, 28; 30$? (You don't need to do the calculation.) Compare this with what we need to know to do multiplication in our base-10 positional number system.

IV. Essay. (40) You have the choice of responding to *either prompt 1 or 2*. State which one you have chosen at the start of your essay.

1) "The distinguishing feature of Babylonian mathematics is its algebraic character." Of the historians we have mentioned, who would agree with this claim, and who would disagree? Explain using the the interpretations your historians would give for the YBC 6967 problem of (what we would phrase as) solving the equation $x = 60/x + 7$.

2) George G. Joseph, the author of *The Crest of the Peacock*, offers this overall evaluation of the ultimate impact of Greek geometry: "There is no denying that the Greek approach to mathematics produced remarkable results, but it also *hampered* [emphasis added] the subsequent development of the subject. ... Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance." First, what does the last sentence mean? What is Joseph getting at? Does this criticism seem to be apt for Diophantus' *Arithmetica*? Then, which side of this debate do you come down on personally? Should all the mathematics we learn and do have practical usefulness or significance?