MONT106Q – Mathematical Thinking
Group Project on Mancala: "The Game Played by Kings and Cowherds"
November 16 and 18, 2016

Background

The name *mancala* is an Arabic word (meaning "transferring") that refers to a whole family of slightly different games. The family is of great antiquity and these games are played all over Africa and the Middle East. It is probably fair to say that these games are analogous to *chess* in European culture – they are seen as both a recreation and an intellectual challenge.

All games in this family, sometimes called "count and capture" games, have the same basic idea. At the start, stones (or seeds, or ...) are placed in a collection of pits. During play, players select a pit and "sow" the stones in that pit into other pits in a definite pattern. In the process stones can be "captured" and placed into a "store." The goal is simple – accumulate more stones in your "store" than your opponent. The variations in

- the layout of the pits,
- the numbers of stones used, and
- the rules by which the game is carried out are almost endless.

We will look at a comparatively basic form, usually called *kalah*. (Some of you may have seen a commercial version of a similar game called Oh-Wah-Ree.)

How kalah is played

Kalah is played on a board with two rows of six pits facing each other. Each player "owns" a row of six pits, plus a "store" to the right of the pits. When the game begins, there are some number of stones in each pit (usually four or six), and the two stores are empty. The players alternate turns.

In each turn a player begins by selecting one of his pits. All of the stones in that pit are picked up and distributed *counter-clockwise* around the board, placing one stone in each pit. Stones are distributed into the player's own "store," but the opponent's store is skipped.

(a) If the last stone distributed lands in the player's own "store," the player gets another turn.
(b) If the last stone lands on an empty pit owned by the player and the opponent's pit directly across the board contains at least one stone, the player takes all the stones in that pit plus the last played stone and puts them all in his or her "store."
(c) Otherwise, the player's turn ends, and the opponent moves.
(d) If all of one player's pits become empty, even if it is not his or her turn to play, the stones remaining in the pits of the opponent are put in the opponent's "store," and the game ends.

(e) The player with the most stones in his or her "store" at the end wins the game.

There is a free version of *kalah* that can be played online at

`http://www.freegaming.de/onlinegames/Puzzle-|-Logic-Games/Mancala.html`

There is also a link to there from the course homepage. You will try this below (although be careful – it can be addictive!).

Comments

If you play *kalah* a few times against the computer, you should quickly come to the conclusion that while the rules are easy and play is simple, understanding who will win from a given situation, and how to play *well* is definitely not simple(!) There are subtleties upon subtleties – just like in chess!

Some special positions

There are a number of special positions from which one of the players can basically force the game to end by playing *all of* the stones from the pits on his or her side of the board into his or her store. For example, suppose the pits contain the following numbers of stones:

$$0, 0, 4, 2, 2, 0$$

left to right. Then the player can sow from the rightmost pit with 2 to reach

$$0, 0, 4, 2, 0, 1.$$

Since the last stone played landed in the store, the player gets another turn and can then continue as follows:

$$0, 0, 4, 2, 0, 0.$$

then

$$0, 0, 0, 3, 1, 1,$$

then

$$0, 0, 0, 3, 1, 0,$$

then

$$0, 0, 0, 0, 2, 1,$$

then

$$0, 0, 0, 0, 2, 0,$$

then

$$0, 0, 0, 0, 0, 1,$$

then finally

$$0, 0, 0, 0, 0, 0.$$

All 8 stones in the player's pits originally have been added to his/her store, and he/she captures all of his/her opponent's stones as well!

As one can guess, recognizing and exploiting these (and related) positions is a key ingredient of *kalah* strategy.

Mancala strategy and mathematics

A) Play several games against the computer using the link from the course homepage. Did you ever win? Did you come close? Can you see elements of the strategy the program is using?

B) Let's say that a pit is "*harvestable*" if sowing from that pit ends in the store. If you analyze the above positions, you will see that we always played from the *rightmost harvestable* pit (when there was a choice – in some cases, there was only one harvestable pit). Explain why this is *always the correct play* if you are in a situation where you can play all of your stones into your store. What happens if there is a choice and you play from a harvestable pit farther to the left?

C) With 6 pits as in standard *kalah*, there are exactly 17 positions as above where you can get *all of your stones to the store* if you play correctly (*without* "wrapping around" to the opponent's side of the board). Find all of them. (Hint: If you think about the example above, you will see that you have, in fact, seen 8 of the 17 already(!))

D) What if you start with more than six pits? Find several additional positions from which you could force the end of the game if you started with stones in a row of 7 pits, 8 pits, and 9 pits.

There's an interesting "tidbit" here. The positions we are looking at are called "winning Tchoukaillon positions" because there is another "solitaire" version of *kalah* by that name. It has been proved mathematically that if you call $s(n) =$ *the smallest number of stones for which a special position of this type using* n *pits exists*, then $s(n)$ grows like $\frac{n^2}{\pi}$ as $n \to \infty$. Who would have thought *mancala* had anything to do with π?!