MONT 105Q – Mathematical Journeys
Midterm Exam, April 1, 2016
Your Name: _____

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra sheet of scratch paper at the end. There are 100 possible points, distributed as indicated in the questions.

I. The following data set has $n = 11$:

$$1.3, \ 2.2, \ 0.8, \ 2.3, \ 3.9, \ 4.1, \ 2.2, \ 1.7, \ 1.2, \ 3.1, \ 2.4$$

(Note: These are *not in numerical order.*)

(A) (5) Find the "5-number" summary for this data set (min, 1st quartile, median, 3rd quartile, max).

(B) (5) Construct a frequency histogram for the data using 5 equal bins on the range 0.0 to 5.0.

(C) (10) Compute the SD of the data set, showing all work to justify your answer. How many of the data values lie within one SD of the mean?

(D) (5) If the numbers in this data set are written on slips of paper and put in a box, the box is shaken, and two slips are drawn at random without replacement, what is the probability that both are < 3.0?

II. Answer each question with a sentence or two. Suppose that a researcher collects 40 tufted titmice (these are small North American songbirds) to measure their body weights. The collected birds have an average weight of 25 g with an SD of 1 g. Think of this as a sampling process.

(A) (5) What is the population? What is the sample?

(B) (5) What could we say about the distribution of sample mean weights of samples of size 100 of tufted titmice (even if the distribution of the individual weights was unknown)? Explain.

(C) (5) The average lifespan of tufted titmice is 2.1 years but many birds die as nestlings (i.e. before reaching maturity, which takes about 4 months from the time they hatch). Would you expect the average age of the birds in a sample of adults collected in the wild to be larger or smaller than 2.1 years? Explain.

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(D) (5) If you knew that the researcher was only taking birds from a particular location, is that a random sample from the appropriate population? Might that process introduce a bias in the measured weights?

III. Suppose that a large data set of standardized raw test scores is *normally distributed* with $\mu = 75$ and $\sigma = 4$. The "z-score" of a raw test score x is computed by

$$z = \frac{x - \mu}{\sigma}.$$

(A) (5) What would be the z-score of a raw test score of 80?

(B) (5) Based on this information, if a raw test score x is selected at random from the data set, what is the probability that $74 \leq x \leq 80$?

(C) (10) Based on this information, if $n = 20$ raw test scores are selected at random, what is the probability that *none of them* is in the range $74 \leq x \leq 80$.

IV. Essay. (35) What is the central thesis of the article *The Dawning of the Age of Stochasticity* by David Mumford? How does Mumford think mathematics should develop in the current century? How is his proposal related to the outcomes of the various attempts to solve the "crisis of foundations" we read about in *Logicomix*? Would Mumford agree or disagree with this statement: "what mathematicians should be doing is closer to the kind of science Darwin practiced on the voyage of the *Beagle* than it is to a Euclidean search for absolute certainty." Explain.