

MONT 104Q – Mathematical Journeys
Midterm Exam, November 2, 2015
Your Name: _____

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra sheet of scratch paper at the end. There are 100 possible points, distributed as indicated in the questions.

I.

- (A) (10) G. H. Hardy included two theorems and proofs from Euclid's *Elements* as prime examples of "real, serious, beautiful mathematics" in his book *A Mathematician's Apology*. Give the statements of those two theorems. *You do not need to supply the proofs for these.*

First theorem:

Second theorem:

- (B) (5) Hardy says "the mathematics which has permanent aesthetic value ... may continue to cause intense emotional satisfaction to thousands of people after thousands of years." About how much time (to within 100 years) separates the time of Euclid from Hardy's own time?

II.

- (A) (10) State the 5 Common Notions (Axioms) and 5 Postulates at the start of Book I of the *Elements* of Euclid.

(B) (5) What is the purpose of the statements from part A in Euclid's logical scheme? How are the Common Notions different from the Postulates?

III. (10) Proposition 1 in Book I of Euclid's *Elements* is a construction for an equilateral triangle with side equal to any given line segment AB. Give the construction and the proof that the construction is valid, with justifications for each of the steps in the proof based on your answers to question II.

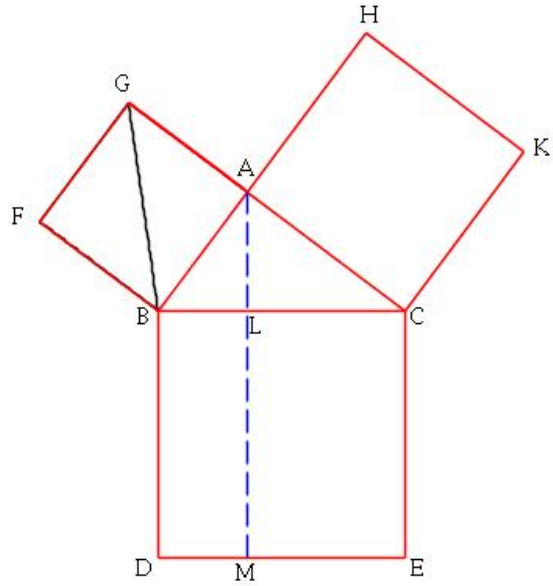


Figure 1: Figure for Proposition 47, Book I

IV. Proposition 47 in Book I of the *Elements* is a famous statement from geometry illustrated by the figure above. Use the labeling here in your answers to all parts.

(A) (5) Give the statement *in Euclid's form* and the usual name of this result.

(B) (5) How is the dotted line AM in the figure constructed?

(C) (5) In the first part of the proof, Euclid shows that $\triangle GBF$ has the same area as what other triangle in the figure? Why does that follow?

(D) (5) The second part of the proof consists of showing that $\triangle FBC$ and $\triangle ABD$ are congruent. How does that follow? (Show that is true using one of the triangle congruence results proved before in Book I.)

(E) (5) How does Euclid conclude that $ABFG$ and $BLMD$ have the same area? And how does he conclude the proof?

IV. Essay. (35) George G. Joseph, the author of an interesting book about the history of mathematics called *The Crest of the Peacock*, offered this overall evaluation of the ultimate impact of Greek geometry: “There is no denying that the Greek approach to mathematics produced remarkable results, but it also hampered the subsequent development of the subject. ... Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance.” First, what does the last sentence mean? What is Joseph getting at? Then, based on what he said in *A Mathematicians Apology*, how would G.H. Hardy respond to Joseph? Finally, which side of this debate do you come down on personally? Should all the mathematics we learn and do have practical usefulness or significance?

