MONT 104Q – Mathematical Journeys
Make-up Midterm Exam, November 5, 2015
Your Name: ________________________

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra sheet of scratch paper at the end. There are 100 possible points, distributed as indicated in the questions.

I.

(A) (10) G. H. Hardy included two theorems and proofs from Euclid's *Elements* as prime examples of "real, serious, beautiful mathematics" in his book *A Mathematician's Apology*. Give the statements of those two theorems.

First theorem:

Second theorem:

(B) (10) Give the proof of either one of the statments from part (A) – your choice, but *you only need to do one.*

(C) (5) Hardy says “the mathematics which has permanent aesthetic value ... may continue to cause intense emotional satisfaction to thousands of people after thousands of years.” About how much time (to within 100 years) separates the time of Euclid from Hardy’s own time?

II.

(A) (10) State the 5 Common Notions (Axioms) and 5 Postulates at the start of Book I of the *Elements* of Euclid.

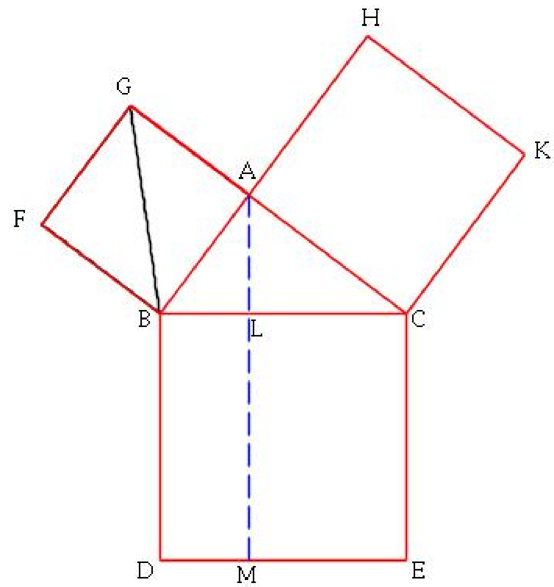

Figure 1: Figure for Proposition 47, Book I

(B) (5) What is the purpose of the statements from part A in Euclid's logical scheme? How are the Common Notions different from the Postulates?

III. Proposition 47 in Book I of the *Elements* is a famous statement from geometry illustrated by the figure above. Use the labeling here in your answers to all parts.

(A) (5) Give the statement *in Euclid's form* and the usual name of this result.

(B) (5) How is the dotted line AM in the figure constructed?

(C) (5) In the first part of the proof, Euclid shows that ΔGBF has the same area as what other triangle in the figure? Why does that follow?

(D) (5) The second part of the proof consists of showing that ΔFBC and ΔABD are congruent. How does that follow? (Show that is true using one of the triangle congruence results proved before in Book I.)

(E) (5) How does Euclid conclude that $ABFG$ and $BLMD$ have the same area? And how does he conclude the proof?

IV. Essay. (35) Peter Rudman, the author of a book about the extent to which the Pythagorean theorem was anticipated by work in the Old Babylonian period (about 2000 - 1600 BCE) wrote this: “High school mathematics education today, with its emphasis on creating high scores on standardized tests, all too often neglects the derivations where mathematics is learned and emphasizes memorizing the equations that provide quick solutions in the standardized tests but that are then rapidly forgotten ... .” What are the “derivations” that Rudman is referring to? Why do you think he says that that “is where mathematics is learned?” How does what he is saying connect with things we read in Hardy’s *Mathematician’s Apology* and that we have done so far in this course? Does his description of high school mathematics match your experience?