I. Short answer. Answer any 6 of the following.

A) (2.5) What base 10 number is represented by these Egyptian hieroglyphs?

Answer: 372

B) (2.5) What base 10 number is represented in the Babylonian way by these symbols?

Answer: $25 \times 60 + 47 = 1547$

C) (2.5) Approximately *when* was the Middle Kingdom period in Egyptian history?

Answer: Approximately 2000 to 1600 BCE

D) (2.5) The name for the writing system used in question B is

Answer: cuneiform

E) (2.5) What are the two most important surviving records of ancient Egyptian mathematics?

Answer: The Rhind and Moscow mathematical papyri

F) (2.5) Most of the geographical area of the Old Babylonian kingdoms is contained in what present-day country?

Answer: Iraq

G) (2.5) Approximately when did Thales live and what part of the Greek world did he come from?

Answer: Approximately 624BCE - 547BCE; Miletus in Asia Minor (present-day Turkey)

H) (2.5) Who founded the Academy in Athens whose entryway was inscribed "let no one unversed in geometry enter here?"

Answer: Plato

II. Compute "in the Egyptian way"

A) (15) 37×136

Solution: Begin with successive doublings until you see the next multiplier of 136 would be larger than 37:

$$\begin{aligned} 1 \times 136 &= 136 - \\ 2 \times 136 &= 272 \\ 4 \times 136 &= 544 - \\ 8 \times 136 &= 1088 \\ 16 \times 136 &= 2176 \\ 32 \times 136 &= 4352 - \end{aligned}$$

($2 \times 32 = 64$ so we don't need to go farther). Then $37 = 32 + 4 + 1$, so $37 \times 136 = 4352 + 544 + 136 = 5032$.

B) (15) $52 \div 18$ ("compute with 18 until you find 52")

Solution: Proceed as in part A to begin:

$$\begin{aligned} 1 \times 18 &= 18 \\ 2 \times 18 &= 36 \end{aligned}$$

(Then $4 \times 18 = 72 > 52$, so we don't need that.) Then $52 - 36 = 16$. So now we start in with fractions of 18. The simplest way to do this to recall that the Egyptians did also allow $\frac{2}{3} = \overline{\overline{3}}$, then successively halve:

$$\begin{aligned} \overline{\overline{3}} \times 18 &= 12 \\ \overline{3} \times 18 &= 6 \\ \overline{6} \times 18 &= 3 \\ \overline{18} \times 18 &= 1. \end{aligned}$$

Then $16 = 12 + 3 + 1$, so $16 \div 18 = \overline{\overline{3}} + \overline{6} + \overline{18}$ and

$$52 \div 18 = 2 + \overline{\overline{3}} + \overline{6} + \overline{18}.$$

III. (20) An Old Babylonian problem text asks for the side of a square if the area of the square minus the side is the base 60 number $14, 30; 0$. The tablet says to do this to solve the problem (all numbers in base 60, of course!): "Take half of 1, which is $0; 30$, square that to get $0;15$, add the $14,30$ to get $14,30;15$. The last number is the square of $29;30$. Now add the $0;30$ to get 30, which is the side of the square."

In modern language, the problem is to solve for x if $x^2 - x - 870 = 0$. Find x by solving this equation, then explain how the Babylonian method of solution is essentially the same as using the quadratic formula for the equation $x^2 - x - 870 = 0$.

Solution: If we apply the quadratic formula to solve the equation, we get

$$x = \frac{1 \pm \sqrt{1 + 4 \cdot 870}}{2} = 30, -29.$$

The negative root is not relevant for the geometry, so we take $x = 30$, which comes with the $+$ sign. Note that result from the quadratic formula can be rearranged as follows

$$30 = \frac{1}{2} + \sqrt{\left(\frac{1}{2}\right)^2 + 870}$$

In modern algebraic notation this gives the same steps as described on the Babylonian tablet: Under the radical, we start by squaring $\frac{1}{2}$, add 870, take the square root, then add $\frac{1}{2}$.

IV. (10) What is Proposition 1 in Euclid's *Elements*? Describe the associated construction.

Solution: The statment is: To construct an equilateral triangle with any given line segment as one side. The construction is as follows. Say the line segment is AB. Using Postulate 3, construct one circle with center at A and radius AB, then a second circle with center at B and radius AB. Let C be one of the intersections of those circles. Then ΔABC is equilateral (since $AB = AC = BC$).

IV. Essay. (25) "The distinguishing feature of Babylonian mathematics is its algebraic character." Of the historians we have mentioned, who would agree with this claim, and who would disagree? Explain using the the interpretations your historians would give for the YBC 6967 problem of solving the equation $x = 60/x + 7$.

Model Answer: The main historian we discussed who would agree with this statement is Otto Neugebauer. His view was that even when a Babylonian problem could be interpreted using geometric ideas, the Babylonians were probably thinking in numerical and algebraic terms. His evidence for saying this was, first, that many of the surviving texts used extensive tables of numerical data (think of the $60/n$ table that we looked at in Discussion 1), and second, that in some cases (as in the problem from question III above), the questions the Babylonians asked often combined lengths and areas in ways that would not really make sense in geometric terms. Neugebauer's explanation for the YBC 6967 problem was that the step-by-step solution given could be explained by using the algebraic identity

$$\left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

with $a = x$ and $b = \frac{60}{x}$. Given that $a - b = 7$, and knowing $ab = 60$, you can use the identity to solve for a and b and that is one way to read what the solution says.

The main historian we discussed who would disagree with this is Jens Høyrup, although Eleanor Robson would also agree with this point of view. Høyrup's main ideas about YBC 6967 were that algebraic interpretations, although perhaps mathematically equivalent to what the Babylonians said, are probably conceptually anachronistic in the sense that the Babylonians just did not have the formalism and the algebraic tools to have approached the problem that way. Moreover, even when we might tend to interpret a Babylonian text in algebraic terms, the actual language they used indicates that they were more likely thinking geometrically. For instance, Høyrup gives a way to understand the YBC 6967 solution as a series of "cut and paste" geometric operations on rectangles.