Your Name: ______________________

Directions

- Do all work on the sheets provided
- If you use the back of a sheet, please place a note telling me to look there.
- There is an extra sheet of scratch paper at the end. You may detach that if you like, but please hand it in with your exam.

Please do not write below on this page.

Problem	Points/Poss
I	/ 50
II	/ 25
III or III'	/ 25
IV	/ 25
V	/ 25
VI (Essay)	/ 50
Total	/ 200

HAPPY HOLIDAYS!

I. Identifications. For any 5 of the following historical figures, give the approximate historical period in which they lived, where they were active, and the main contributions they made that we discussed. (If you submit answers for more than 5, only the best 5 will be counted.)

(A) (10) Eratosthenes

(B) (10) Claudius Ptolemy

(C) (10) Muhmammad ibn Musa al-Khwarizmi

(D) (10) Thabit ibn Qurra

(E) (10) Leonardo Pisano ("Fibonacci")

(F) (10) Filippo Brunelleschi

II.

(A) (10) What are the 5 Common Notions (Axioms) and 5 Postulates that appear at the beginning of Book I of Euclid's *Elements*?

(B) (15) Proposition 32 in Book I deals with the sum of the angles in a triangle. What is the exact statement that Euclid proves? Outline the construction and proof that establishes this Proposition. (You don't need to quote earlier propositions by number, but do give a reason for each of your assertions.)

Choose either this problem (III) or III' on the next page. (If you submit solutions for both, only the better of the two will be used.)

(A) (15) What is true about the sum of the angles in a triangle under the alternate *hyperbolic* form (H) of Postulate 5? Outline the proof, assuming the needed properties of Saccheri quadrilaterals.

(B) (10) Was the result in part A the contradiction that Girolamo Saccheri, S. J. was seeking under the assumption (H)? Explain. What did Janos Bolyai and Nikolai Ivanovich Lobachevsky eventually show about the search for a contradiction starting from (H)?

III'. (If you submit solutions for both this problem and III on the last page, only the better of the two will be used.)

(A) (15) The following problem appears in a text by the Indian mathematician Mahavira (about 850 C.E.). Solve it using our modern notation and algebraic methods: Of a collection of mangoes, the king took $1/6$, the queen $1/5$ of the remainder, the chief princes $1/4, 1/3, 1/2$ of the successive remainders, and the youngest child took the remaining 3 mangoes. Oh you who are clever in miscellaneous problems on fractions, give the measure (number) of the original collection of mangoes.

(B) (10) What was probably the single largest contribution of ancient Indian mathematics to the way we do mathematics today? How and when did this contribution enter our mathematical tradition?

IV.

A) (15) Draw the diagram for the Chinese "*go gou* theorem" (with a general right triangle, sides a, b and hypotenuse c) and show how it provides an algebraic/geometric proof of the general "Pythagorean theorem."

B) (10) How is this proof different from the one given by Euclid in Proposition 47 of Book I of the *Elements*?

V.

A) (15) What are *perfect numbers*? What are *amicable pairs of numbers*? Give an example of a perfect number and an amicable pair and explain how they satisfy the definitions.

B) (10) How does the historical development of knowledge about these number-theoretical ideas tie together the Greeks, the medieval Islamic mathematicians, and mathematics in the contemporary world?

VI. Essay. (50) Here are two contrasting statements about the ultimate legacy of Greek mathematics:

- "The death of Archimedes by the hands of a Roman soldier is symbolical of a world-change of the first magnitude: the Greeks, with their love of abstract science, were superseded ... by the practical Romans. The Romans were a great race, but they were cursed with the sterility which waits upon practicality. They did not improve upon the knowledge of their forefathers, and all their advances were confined to the minor technical details of engineering. They were not dreamers enough to arrive at new points of view ... No Roman lost his life because he was absorbed in the contemplation of a mathematical diagram." (Alfred North Whitehead – 20th century philosopher)

- "There is no denying that the Greek approach to mathematics produced remarkable results, but it also hampered the subsequent development of the subject. ... The Greek preoccupation with geometry ... was a serious constraint. Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance." (George G. Joseph, *The Crest of the Peacock*)

Begin by summarizing and explaining your understanding of each statement. Then address the following questions: Have we seen examples in this class where practicality was *not sterile*, where concern with applications actually enriched the history of mathematics and made further progress possible? Have we seen other examples where the "abstract idealized constructs" of pure mathematics led to major advances? Why do you suppose that Joseph doesn't mention Archimedes? Does that omission weaken his point?