MONT 100N – Modeling the Environment
Solutions for Mathematical Questions from Midterm Exam – October 27, 2017

I. The following table gives amounts of carbon (in units of $10^{15}$ kilograms) contained in the major "reservoirs" of this element on planet Earth:

| **Reservoir** | **Carbon Content** |
|---|---|
| Atmosphere | .59 |
| Crust (as fossil fuels) | 3.7 |
| Vegetation | 2.3 |
| Shallow Ocean | .9 |
| Deep Ocean | 37.3 |

A. What percent of the total carbon present on Earth is contained in each of these reservoirs?

*Solution:* The sum of these amounts is $44.79 \times 10^{15}$ kilograms. So, the percentage of the carbon in the deep ocean reservoir, for instance, is

$$\frac{37.3}{44.79} \times 100\% \doteq 83.3\%$$

The other values are computed similarly. Rounding to 1 decimal place,

| **Reservoir** | **Percent of Total Carbon** |
|---|---|
| Atmosphere | 1.3% |
| Crust (as fossil fuels) | 8.3% |
| Vegetation | 5.1% |
| Shallow Ocean | 2.0% |
| Deep Ocean | 83.3% |

B. Construct and draw a chart (your choice of type) showing how the total carbon breaks down into these categories.

*Solution:* Either a pie chart or a bar chart is OK here. If you use a pie chart, the percents above correspond to the fractions of the pie. If you use a bar chart, it would be best to make the vertical axis scale correspond to the actual amounts of carbon (in units of $10^{15}$ kg), so use something like a vertical scale of 0 to 40. However, a scale where the vertical axis shows the percents is also acceptable.

II. Burning coal provides between 9500 and 14000 BTUs of heat energy per pound. Using the table below, answer questions A and B.

A. (8) Express the heat energy from burning one pound of coal as a range of values in calories.

*Solution:* There are .252 Kilocalories in one BTU. Hence, burning one pound of coal yields between

$$9500 \text{ BTU} \times .252 \frac{\text{Kcal}}{\text{BTU}} \doteq 2.39 \times 10^3 \text{ Kcal}$$

and

$$14000 \; BTU \times .252 \frac{\text{Kcal}}{\text{BTU}} \doteq 3.53 \times 10^3 \text{ Kcal.}$$

B. (8) What is the range of heat energy values provided by burning 1 kilogram of coal, expressed in BTUs?

*Solution:* From given information, one pound is .454 kg, so 1 kg $= \frac{1}{.454} \doteq 2.20$ lb. Hence burning 1 kg of coal provides between

$$9500 \text{ BTU/lb} \times 2.20 \text{ lb} = 2.09 \times 10^4 \text{ BTU}$$

and

$$14000 \text{ BTU/lb} \times 2.20 \text{ lb} = 3.08 \times 10^4 \text{ BTU.}$$

C. (4) The percentage difference is

$$\frac{14000 - 9500}{9500} \times 100\% \doteq 47.3\%$$

(the anthracite coal generates 47.3% more heat than the bituminous).

IV. (15) The Honda Civic comes in a standard (gasoline engine) version and a hybrid (gasoline-electric) version. The standard version has a fuel efficiency of about 34 miles per gallon, while the hybrid version gets about 44 miles per gallon. The hybrid version had a price about \$4000 more than the standard. If you bought the hybrid and drive about 10000 miles per year, about how many years will it take for your savings in gasoline costs to make up for the difference in price? Assume that gasoline will average \$2.00 per gallon over the life of the vehicle.

*Solution:* If you drive 10000 miles with the hybrid version, then you use $10000 \times \frac{1}{44} \doteq 227.3$ gallons of gas in one year and you spend $227.3 \times \$3.00 \doteq \$681.90$ on gasoline. With the standard version you use $10000 \times \frac{1}{34} \doteq 294.1$ gallons of gasoline and you spend \$882.30. The difference is \$200.40. To make up the \$4000 greater cost of the hybrid model, you would need to keep the hybrid for about $\frac{4000}{200.40} \doteq 20$ years. (Comment: These are pretty realistic numbers, and this is one reason why hybrids are still seen as less attractive options by many people.)

IV. Consider the Excel scatter plot given in the exam. Place a check next to the best responses to the questions below.

1. (5) The least squares regression line for this data set would have a slope $m$ about:

$m = -.1$ ____X____

$m = .3$ _____

$m = 1$ _____

The best answer is about $m = -.1$ since the slope definitely should be negative and the velocity is decreasing by roughly 1.4 feet per second over the roughly 11 feet change in depth. The actual slope is $m \doteq -0.12$.

2. (5) Based on the scatter plot, the $R^2$ statistic is probably:

between .8 and 1 ____X____

between .4 and .6 _____

between .2 and .4 _____

between .0 and .2 _____

The best answer is $.8 < R^2 < 1$ since the data is relatively linear (so $R^2$ will be close to 1). The actual value is $R^2 \doteq .88$.