**College of the Holy Cross, Fall Semester, 2017**
**MONT 100N – Modeling the Environment, Final Exam**
**Friday, December 15**

**Your Name**: _____

**Instructions:** Please show all work necessary to justify your answers, and write your answers in the spaces provided. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

| Problem | Points/Poss |
|---------|-------------|
| I       | / 20        |
| II      | / 25        |
| III     | / 15        |
| IV      | / 10        |
| Essay   | / 30        |
| Total   | /100        |

*Have a peaceful and joyous holiday season!*

I. In 1990, forests covered $4.047 \times 10^9$ hectares of the Earth's surface. By 2000, forest area had decreased to $4.038 \times 10^9$ hectares. Assuming that the decrease in forest area is linear, and that it will continue at the same rate into the future, in this problem you will develop a linear model for the forest area remaining as a function of $t =$ years since 1990.

A. (5) Determine the slope for the linear model of the forest area.

B. (5) What is the linear equation modeling the forest area as a function of $t =$ years since 1990.

C. (5) According to your model, in what year will the forest area reach $4.0 \times 10^9$ hectares?

D. (5) According to the United Nations Food and Agriculture Organization, the actual forest area remaining in 2010 was $4.033 \times 10^9$ hectares. How close is the prediction your model from part B gives for the forest area in 2010?

II. Wind power has emerged as a fast growing source of energy for electrical power generation in recent years. In 2016, the generating power of wind turbines installed around the world was about 301 gigawatts and it was increasing at about 33.2% per year.

A. (5) The typical English unit of power is the horsepower. 1 horsepower $= 7.457 \times 10^{-7}$ gigawatts. Convert 301 gigawatts to the equivalent number of horsepower.

B. (5) Construct an exponential model for $WP =$ wind power generation as a function of $t =$ years after 2016. *Use units of $10^2$ gigawatts for WP* – see the entry for 2016 in the table below.

C. (10) Fill in the table of values for $WP$ below with values predicted by your model for the years $2017 - 2022$. Round to 2 decimal places. About how many years will it take for $WP$ to reach approximately double the 2016 level?

| $Year$ | 2016 | 2017 | 2018 | 2019 | 2020 | 2021 |
|--------|------|------|------|------|------|------|
| $WP$   | 3.01 |      |      |      |      |      |

D. (5) How many years will it take for wind power generation to reach $20 \times 10^2$ gigawatts according to your model?

III. Suppose that a population of fast-reproducing insects in an area has a natural growth rate of 5% per month from births and deaths, and that there is a net migration *loss* of 20 individuals per month.

A. (5) Which of the following difference equation models for $P(n) =$ population in month $n$ fits the description above? (Place a check next to the correct one.)

1) _____ $P(n) = 5P(n-1) + 20$

2) _____ $P(n) = 1.05P(n-20)$

3) _____ $P(n) = 1.05P(n-1) - 20$

4) _____ $P(n) = 1.05P(n-1) + 20$

B. (5) Using an initial value $P(0) = 500$, determine the populations in months $1, 2, 3, 4, 5$ according to the model you picked in part A and record the values in the following table (round any decimal values to the nearest whole number)

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $P(n)$ | 500 | | | | | |

C. (5) What happens to the population in the long run as $n$ increase? Does it tend to a definite value?

IV. Answer *any two* of the following three questions (only the best two will be counted if you answer more than two).

A. (5) If you are fitting an exponential model to a data set $(x_i, y_i)$ "by hand," you start by transforming the data to $(X, Y) = (x_i, \log(y_i))$. If the best fit regression line for the transformed data is $Y = mX + b$, what is the corresponding exponential model? (Assume the logarithms have base 10 as we discussed in class.)

B. (5) What does the $R^2$ statistic in measure in linear regression? How did we use it? Explain what it would mean, for instance if $R^2 = 1$.

C. (5) What type of chart (scatterplot, pie chart, bar chart, etc.) would be most useful to describe the composition of a forest if there 5 different types of trees present in different concentrations per acre? Explain, and illustrate your answer with a chart if a typical acre of forest contains 10 oaks, 12 maples, 5 pines, 2 hemlocks, and 1 chestnut.

*Essay* (30) You have the choice of answering either one of the following prompts. Please clearly indicate which prompt you are choosing.

Topic A In general terms, what is a mathematical model? Describe in general terms what they are, how they are constructed, and how they are used. Give examples of two different types of mathematical models we have studied in this course. Even if mathematical models don't capture every feature of a real world situation, why is it still important to develop them and understand the information we get from them? As an example, why is it important to understand how radioactive substances decay? What types of models that we discussed would apply to describe that process? What are three of the main issues discussed in the film *Containment* that we saw with the rest of the Natural World Cluster? How do those issues connect with other topics we discussed semester like the properties of different sources of energy used in the U.S. and their effects on the environment? How are mathematical models important in understanding our choices of which energy sources to use?

Topic B What does Jared Diamond mean by a "collapse" in his book with that name? Discuss the five-point framework for understanding causes of collapses that Diamond lays out in the Prologue. Which of those factors does he say are *always* present? Discuss how the framework applies to *either* the collapse of the Easter Island civilization or the Classic Maya civilization in Central America. "The past is a foreign country; they do things differently there," is the famous opening line of a novel called *The Go-Between* by L. P. Hartley. How does Diamond's point of view in *Collapse* relate to that attitude about the past? For instance, would he say the past is not really that foreign after all, or would he argue that there is some key difference that sets us apart from previous civilizations and lessens the causes for concern about our ultimate fate? What do *you* think about this question, after having read several sections of *Collapse*?