College of the Holy Cross, Fall Semester 2017
MATH 243 – Mathematical Structures, section 2
Exam 3 – December 7

Your Name: _____

Instructions Please write your answers in the spaces provided on the following pages, and show work on the test itself. *For possible partial credit, even if you cannot completely solve a problem, include definitions of terms involved, partial results you can do, etc.* Use the back of the preceding page if you need more space for scratch work.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 20
III	/ 20
IV	/ 15
V	/ 20
Total	/100

I. (25) Give the statement and proof of "Fermat's Little Theorem."

II. (20) An RSA public key encryption system has public key $m = 551, e = 11$. "Crack the code" by determining the private key information: p, q, d.

III. Let $f : A \to B$ be a mapping.

(A) (10) Show that if U_1, U_2 are subsets of B, then $f^{-1}(U_1 \cap U_2) = f^{-1}(U_1) \cap f^{-1}(U_2)$.

(B) (10) If f is injective, and T_1, T_2 are subsets of A, show that $f(T_1) \cap f(T_2) \neq \emptyset$ implies $T_1 \cap T_2 \neq \emptyset$.

IV. (15) Let R be the relation on $\mathbb{R}\backslash\{0\}$ defined by $a\ R\ b \Leftrightarrow \frac{a}{b} \in \mathbb{Q}$. See below.[1] Is R an equivalence relation? Prove your assertion.

[1]Here $\mathbb{Q} = \{m/n : m, n \in \mathbb{Z}, n \neq 0\}$ is the set of rational numbers.

V.

(A) (10) Show that $\mathbb{N}$ is not bounded above in the real numbers.

(B) (10) Use part (A) to show that for all real numbers $\varepsilon > 0$, there exist $n \in \mathbb{N}$ such that $\left|1 - (1 + \frac{(-1)^n}{\sqrt{n}})\right| < \varepsilon$