College of the Holy Cross, Fall Semester 2017
MATH 243 – Mathematical Structures, section 2
Exam 2 – November 2

Your Name: _____

Instructions Please write your answers in the spaces provided on the following pages, and show work on the test itself. *For possible partial credit, even if you cannot completely solve a problem, include definitions of terms involved, partial results you can do, etc.* Use the back of the preceding page if you need more space for scratch work.

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 35
III	/ 15
IV	/ 15
V	/ 15
Total	/100

I. Let $f : \mathbb{Z}/29\mathbb{Z} \to \mathbb{Z}/29\mathbb{Z}$ be the mapping defined by $f([x]) = [x] + [12]$.

(A) (10) Show that f is injective.

(B) (10) Is f surjective? Why or why not?

II.

(A) (20) Give a precise statement of the Division Algorithm in $\mathbb{Z}$, and prove *both* the Existence and Uniqueness parts.

(B) (15) Use the Euclidean algorithm to find the integer $d = \gcd(585, 108)$ and express d in the form $d = m \cdot 585 + n \cdot 108$ for some integers m, n.

III. (10) Let a, b, c be integers. Show that if $\gcd(a, b) = 1$ and $a|(bc)$, then $a|c$.

IV. (15) Find a solution x of the congruence $31x \equiv 6 \bmod 64$ with $0 \leq x < 64$.

V. (15) Construct the *multiplication* table for $(\mathbb{Z}/12\mathbb{Z})^{\times}$.