

College of the Holy Cross, Fall Semester, 2017
MATH 243, Section 2 – Midterm 1
Friday, September 29

Your Name: _____

Instructions Please write your answers in the spaces provided, and show work on the test itself. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

Please do not write in the space below

Problem	Points/Poss
1	/ 20
2	/ 20
3	/ 20
4	/ 20
5	/ 20
Total	/100

1. (a) (15) Construct the truth table for the statement

$((\text{not } Q) \text{ and } (P \text{ implies } Q)) \text{ implies } (\text{not } P)$

- (b) (5) Suppose you know that $x \in \mathbf{R}$ implies that $x^2 \geq 0$ and someone one hands you a mathematical object x for which you can define x^2 , and for which x^2 is a real number with $x^2 < 0$. What can you conclude?

2. Let

$$A = \{x \in \mathbb{Z} : x = 3k + 1, \text{ some } k \in \mathbb{Z}\}$$

$$B = \{x \in \mathbb{Z} : x = 4k, \text{ some } k \in \mathbb{Z}\}$$

$$C = \{-9, -8, -7, -6, -5, -4, -3, -2, -1\}.$$

(a) (7) What is the set $(A \cup B) \cap C$?

(b) (7) What is the set $B^c \cap C$?

(c) (6) Prove or disprove: $4A \subseteq A \cap B$, where $4A = \{4x : x \in A\}$.

3. Consider the statement about $x \in \mathbb{Z}$: If $x^2 \notin 1 + 4\mathbb{Z}$, then x is even.

(a) (10) What is the contrapositive form of this implication?

(b) (10) Prove the contrapositive form. What does this imply about the original statement?

4. (20) Let

$$A = \{x \in \mathbb{Z} : x = -2 + 5k \text{ for some } k \in \mathbb{Z}\}$$

$$B = \{x \in \mathbb{Z} : x = 3 + 5k \text{ for some } k \in \mathbb{Z}\}.$$

Is $A = B$? Prove your assertion.

5. (20) Prove by mathematical induction: If a, d are any constants and $n \geq 0$ is a natural number, then

$$a + (a + d) + (a + 2d) + \cdots + (a + nd) = \frac{n + 1}{2} \cdot (2a + nd).$$