

Math 131 - section 01 - Precalculus Diagnostic Quiz Answers
August 31, 2007

Circle the correct answer. **Show your work.** Please turn over for problems 4,5,6.

1. Simplify: $\sqrt{15} \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right)$

A. $2\frac{\sqrt{15}}{\sqrt{8}}$ B. $\frac{\sqrt{15}}{\sqrt{3} + \sqrt{5}}$ C. 1 D. $\sqrt{3} + \sqrt{5}$ E. $2\sqrt{3}$.

- *Solutions:* The answer is D. There are several ways to do this, but all rely on the fact that $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ for all positive real a, b .
- Method 1: Multiply through:

$$\begin{aligned}\sqrt{15} \left(\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \right) &= \frac{\sqrt{15}}{\sqrt{3}} + \frac{\sqrt{15}}{\sqrt{5}} \\ &= \frac{\sqrt{5}\sqrt{3}}{\sqrt{3}} + \frac{\sqrt{5}\sqrt{3}}{\sqrt{5}} \\ &= \sqrt{5} + \sqrt{3}\end{aligned}$$

- Method 2: Put the terms on the inside the parentheses over a common denominator:

$$\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} = \frac{\sqrt{5} + \sqrt{3}}{\sqrt{15}}$$

Then multiplying by $\sqrt{15}$ gives $\sqrt{5} + \sqrt{3}$ again.

2. Simplify: $\frac{12x}{3x-6} \cdot \frac{x^2-4}{2x+4}$

A. $\frac{x^2 + 12x - 4}{5x - 2}$ B. $2x$ C. $-\frac{4}{3}(x-1)$ D. $\frac{2x(x-2)}{x+2}$ E. $\frac{12x^3 - 48x}{6x^2 - 12}$

- *Solution:* The answer is B. Recall the difference of squares factorization: $x^2 - a^2 = (x + a)(x - a)$. Here, $x^2 - 4 = (x + 2)(x - 2)$, so

$$\frac{12x}{3x - 6} \cdot \frac{x^2 - 4}{2x + 4} = \frac{12x(x + 2)(x - 2)}{3(x - 2) \cdot 2(x + 2)}$$

Cancelling common factors top and bottom yields $2x$.

3. Simplify: $(u^{-5}v^2)^3 \left(\frac{v^2}{u}\right)^{-1}$

- A. $u^{-16}v^8$ B. $u^{-14}v^4$ C. $u^{-9}v^3$ D. $(uv)^{-7}$ E. $u^{-4}v^2$

- *Solution:* The answer is B. Recall the rules for exponents:

$$a^b \cdot a^c = a^{b+c}, \quad (a^b)^c = a^{b \cdot c}, \quad a^{-1} = \frac{1}{a}.$$

Then

$$(u^{-5}v^2)^3 \left(\frac{v^2}{u}\right)^{-1} = u^{-15}v^6 \cdot uv^{-2} = u^{-14}v^4.$$

4. If $f(x) = 5x^2 - 11$, what is $f(a - 2)$?

- *Solution:* Substitute $a - 2$ for the x in the formula to yield

$$f(a - 2) = 5(a - 2)^2 - 11 = 5(a^2 - 4a + 4) - 11 = 5a^2 - 20a + 9.$$

5. Find all values of x satisfying $2(x - 2) > 5$.

- *Solution:* All real $x > 9/2$ (since $x - 2 > 5/2$, so $x > 5/2 + 2 = 9/2$).

6. Solve for x : $2x^2 - x - 6 = 0$ (find all solutions).

- *Solution:* This quadratic factors as $(2x + 3)(x - 2) = 0$, so $2x + 3 = 0$ or $x - 2 = 0$. Hence $x = -3/2$ or $x = 2$. The equation can also be solved by the quadratic formula:

$$x = \frac{1 \pm \sqrt{1 + 49}}{4} = 2, -3/2.$$