College of the Holy Cross, Spring 2008
Math 132, Midterm Exam 3 (All Sections)
Wednesday, April 23, 7 PM

Your Name: ______________________________

Your Section:

Little (8:00am) ____ Ballantine (9:00am) ____
DeStefano (10:00am) ____ DeStefano (noon) ____

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

Problem	Points/Poss
I	/ 15
II	/ 15
III	/ 15
IV	/ 15
V	/ 15
VI	/ 10
VII	/ 15
Total	/100

I.

A. [10 points] Circle the number of the plot showing the direction field for each of the following differential equations. (Note that there are only 3 plots, so the correct answer for one is "None.")

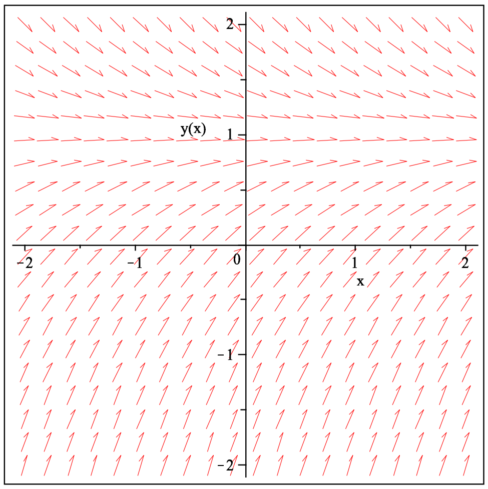
(1) $y' = x$ I II III None

(2) $y' = 1 + y^2$ I II III None

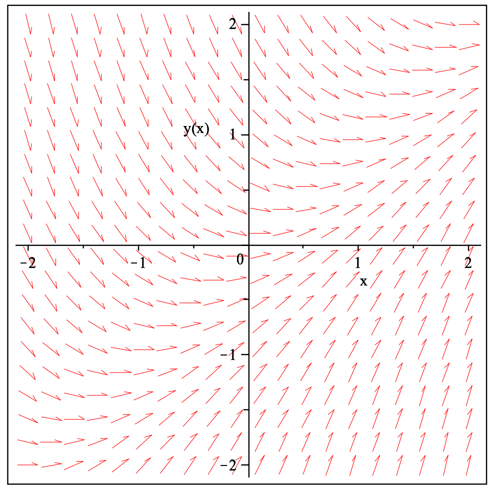
(3) $y' = 1 - y$ I II III None

(4) $y' = x - y$ I II III None

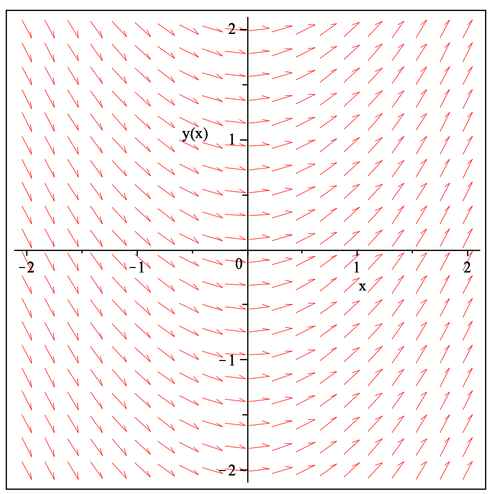
I:



II:



III:



B. [5 points] On the plot for the equation $y' = 1 - y$ from (3) of part A, give a qualitative sketch of the graph of the solution satisfying the initial condition $y(-1) = 0$. Show as much of the graph as you can for both positive and negative x.

II. All parts of this problem deal with the differential equation $y' = 7 - y$.

A. [4 points] Use 4 steps of Euler's method to approximate the solution of this equation with the initial condition $y(0) = 4$ at $x = 2$.

B. [6 points] Find the general solution $y(x)$ of the equation by separating variables and integrating.

C. [5 points] Find the particular solution $y(x)$ satisfying the initial condition $y(0) = 4$ and compute the exact value of $y(2)$.

Particular solution: ______

$y(2) =$ ______

Go to next page.

III. All parts of this question deal with the infinite series

$$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \cdots .$$

A. [4 points] Find a general formula for the nth term of the series as a function of n and write the series in summation ("sigma") notation.

Series =

B. [4 points] Call the nth term in the series a_n. What is $\lim_{n\to\infty} a_n$?

Limit =

C. [4 points] Write out the first 3 *partial sums* of the series.

$s_1 =$

$s_2 =$

$s_3 =$

D. [3 points] Does this series converge or diverge? Explain your answer.

Answer:

IV.

A. [5 points] Does the series $\sum_{n=0}^{\infty} \frac{(-1)^n e^n}{\pi^n}$ converge or diverge? If it is convergent, say why and find the sum; if it is not convergent say why not.

Answer:

B. [10 points] Explain why the Integral Test can be applied to the series $\sum_{n=1}^{\infty} \frac{n}{e^{3n}}$ and use it to determine if the series converges or diverges.

Answer:

V. All parts of this question refer to the power series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt{n}}.$$

A. [9 points] Use the Ratio Test to determine the radius of convergence.

Radius of convergence:

B. [6 points] Test convergence at the endpoints of the interval from part A to determine the interval of convergence. Explain your conclusions.

Interval of convergence:

VI. [10 points] Find the Taylor polynomial of degree $n = 3$ for $f(x) = 2 + 3x + x^3$ at $a = 1$.

Taylor Polynomial:

VII.

A. [5 points] Starting from the Taylor series for $\sin(x)$ at $a = 0$, find a series representation for $f(x) = \dfrac{\sin(x) - x}{x^3}$. Give the first three nonzero terms in your series.

Series for $f(x)$:

B. [5 points] Use your answer in part A to determine $\lim_{x\to 0} \frac{\sin(x) - x}{x^3}$.

Limit: $\boxed{}$

C. [5 points] Use your answer in part A to determine the first three nonzero terms in a series representing the function

$$F(x) = \int_0^x \frac{\sin(t) - t}{t^3}\, dt.$$

Series: $\boxed{}$