Mathematics 131 – Calculus for Physical and Life Sciences 1
Precalculus Diagnostic Quiz – September 7, 2004

Solutions

1. Solve for x: $3x^2 - 21x + 30 = 0$.

Method 1 (easier): Factoring, we see $3x^2 - 21x + 30 = 3(x-2)(x-5)$. This equals zero only when one of the factors is zero. $3 \neq 0$, so the roots are given by

$$\begin{aligned} x - 2 = 0 &\Rightarrow x = 2, \text{ or} \\ x - 5 = 0 &\Rightarrow x = 5. \end{aligned}$$

Method 2 (more calculation): Use the quadratic formula: $ax^2 + bx + c = 0$ has roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here $a = 3, b = -21, c = 30$, so

$$x = \frac{21 \pm \sqrt{441 - 4(3)(30)}}{6} = \frac{21 \pm \sqrt{81}}{6} = \frac{21+9}{6}, \frac{21-9}{6} = 5, 2.$$

Note: You could also factor out the 3, *then* apply the quadratic formula.

2. Let $f(x) = \sqrt{x-1}$ and $g(x) = x^3 + 1$. Find $f(g(x))$? What is the domain of $f(g(x))$?

$$f(g(x)) = \sqrt{g(x) - 1} = \sqrt{(x^3+1) - 1} = \sqrt{x^3} = x^{3/2}$$

The domain of a function defined this way is taken to be the biggest set of real numbers you can substitute into the formula and get a well-defined result. Here, we can't take the square root of a negative number and get a real value, so $x^3 \geq 0$ is necessary. This is the same as $x \geq 0$. The domain is the set of all $x \geq 0$ in the real numbers.

3. Simplify and express as a single fraction:

$$\frac{\frac{x}{\sqrt{x+1}} + \sqrt{x+1}}{\frac{x}{x^2+3}}$$

First we rewrite the fraction as multiplying by the reciprocal of the denominator, then put the terms in the numerator over a common denominator so they can be added, then finally combine:

$$\begin{aligned}
\frac{\frac{x}{\sqrt{x+1}}+\sqrt{x+1}}{\frac{x}{x^2+3}} &= \left(\frac{x}{\sqrt{x+1}}+\sqrt{x+1}\right)\cdot\left(\frac{x^2+3}{x}\right)\\
&= \left(\frac{x}{\sqrt{x+1}}+\frac{x+1}{\sqrt{x+1}}\right)\cdot\left(\frac{x^2+3}{x}\right)\\
&= \left(\frac{2x+1}{\sqrt{x+1}}\right)\cdot\left(\frac{x^2+3}{x}\right)\\
&= \frac{(2x+1)(x^2+3)}{x\sqrt{x+1}}
\end{aligned}$$

(Many other forms possible too.)

4. Let $f(x) = x^2 + 2x$. Compute and simplify: $f(2+h) - f(2-h)$.

In an expression like $f(2+h)$, we take the "inside" (the $2+h$) and substitute that in for x everywhere in the definition of the function:

$$f(2+h) - f(2-h) = [(2+h)^2 + 2(2+h)] - [(2-h)^2 + 2(2-h)]$$

Now expand, collect like powers and simplify:

$$\begin{aligned}
[(2+h)^2 + 2(2+h)] - [(2-h)^2 + 2(2-h)] &= [4 + 4h + h^2 + 4 + 2h] - [4 - 4h + h^2 + 4 - 2h]\\
&= [8 + 6h + h^2] - [8 - 6h + h^2]\\
&= 12h
\end{aligned}$$