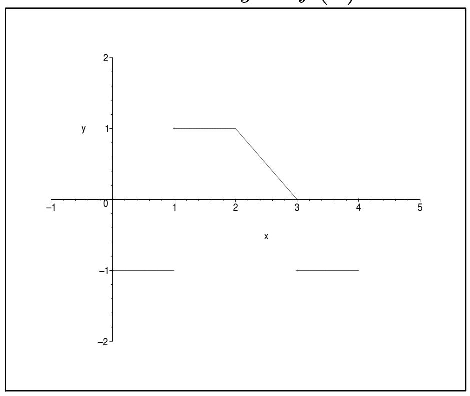
Holy Cross College, Spring Semester, 2005
MATH 132, Section 01, Final Exam
Tuesday, May 10, 2:30 PM
Professor Little

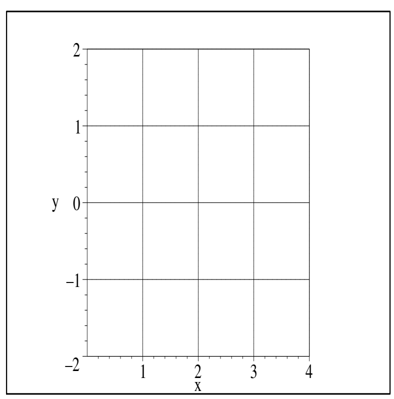
1. The following graph shows the function $y = f(x)$.



   (a) [10 points] Let $F(x)$ be the antiderivative of $f(x)$ with $F(0) = 0$. Compute the entries in the following table of values:

x	0	1	2	3	4
$F(x)$	0				

   (b) [10 points] Sketch the graph $y = F(x)$ on the interval $0 \le x \le 4$:



2. Integrate each of the following. You must show all work and cite any table entries you use for full credit.

   (a) [5 points] $\displaystyle\int \frac{x^2 + 3x + 5}{x^{3/2}}\, dx$

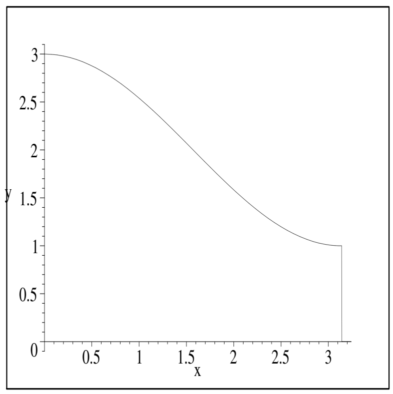
(b) [10 points] $\displaystyle\int \frac{e^t+1}{(e^t+t)^2}\,dt$

(c) [10 points] $\displaystyle\int \frac{dx}{(9-x^2)^{3/2}}$ (use a trigonometric substitution)

(d) [10 points] $\displaystyle\int_0^1 x^2e^{2x}\,dx$

(e) [10 points] $\displaystyle\int \frac{(x-1)\,dx}{x^3+25x}$

3. All parts of this problem refer to the region R in the plane bounded by $y = 2+\cos(x)$, $y = 0$, $x = 0$, and $x = \pi$, shown below:



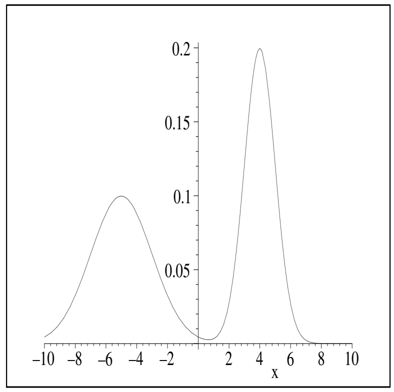
(a) [10 points] Find the volume of the solid with base R whose cross-sections by planes perpendicular to the x-axis are squares.

(b) [10 points] True or False: The solid obtained by rotating R about the x-axis has volume π times the volume from part a. (For full credit, give the integral that would compute the volume of this solid of revolution, and answer the question.)

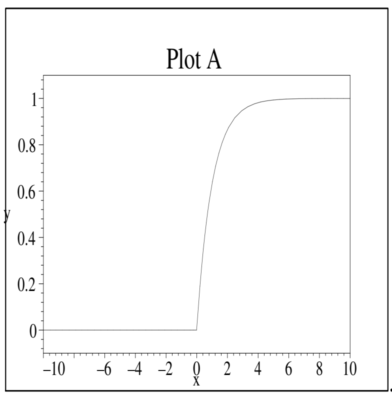
(c) [15 points] A thin metal plate has the shape of the region R (units in meters) and constant mass density $1kg/m^2$. Find the x-coordinate of its center of mass.

(d) [10 points] Set up, *but do not evaluate* the integral giving the arclength of the top edge of R.

4. The following graph shows a probability density function (pdf) for a quantity x:



(a) [5 points] Which of the following two plots shows the corresponding cumulative distribution for x:



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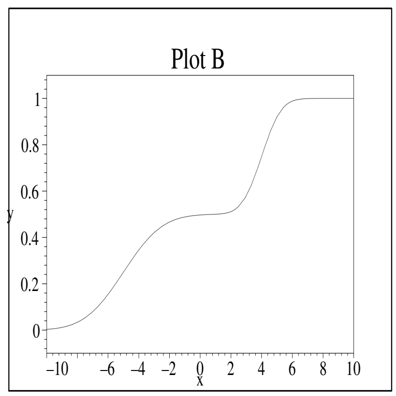

(b) [5 points] Using the appropriate graph, estimate the *median* of x.

5. Does each of the following series converge? For full credit, you must justify your answer completely by showing how the indicated test leads to your stated conclusion.

(a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ – alternating series test.

(b) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{9/10}}$ – integral test

(c) [5 points] $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{\pi^n}$ – any applicable method

6. Using the ratio test, determine the radius of convergence of the following power series

$$\sum_{n=0}^{\infty} \frac{(x+2)^n}{n^2 4^n}$$

([15 points]).

7. All parts of this problem refer to $f(x) = \cos\left(\frac{x}{2}\right)$.

    (a) [15 points] Using the definition of Taylor polynomials, compute the Taylor polynomial of degree $n = 6$ for $f(x)$ at $a = 0$.

    (b) [10 points] Use your answer from part a to approximate $\cos(1.2)$.

8. When a course ends, it's an unfortunate fact of life that students start to forget the material they have learned. One mathematical model for this process states that *the rate at which a student forgets material is proportional to the difference between the material currently remembered and some positive constant* $a < 1$. Let y be the fraction of the original material remembered t weeks after the course has ended. Then $y(0) = 1$, and the model assumes $y(t) > a$ for all t.

    (a) [5 points] Which of the following differential equations is the correct translation of the model described above:

$$I: \frac{dy}{dt} = y - a \qquad II: \frac{dy}{dt} = \frac{-k}{y-a}$$
$$III: \frac{dy}{dt} = k(y - a) \qquad IV: \frac{dy}{dt} = t(y - a).$$

    (b) [10 points] Solve the equation you selected in part a by separation of variables.

    (c) [10 points] Suppose $a = .2$, and after 3 weeks the student only remembers 80% of the material from the course (that is, $y(3) = .8$). How long will it be before the student only remembers one half of the material?