College of the Holy Cross, Spring Semester, 2005
Math 132 Review Sheet for Midterm 1

The first midterm will be held the evening of Wednesday, February 23, from 6:00–7:30. Material from Section 5.4 (properties of the integral) through Section 7.4 (partial fractions and trig substitution) will be covered.

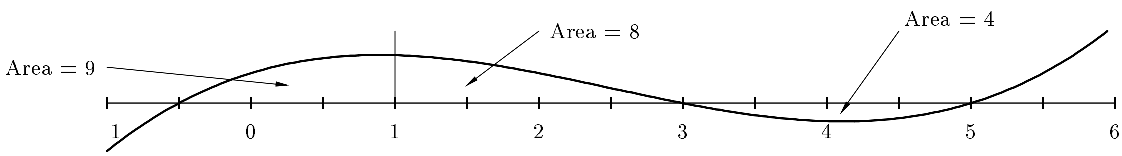
1. At t seconds after liftoff, a rocket experiences acceleration $a = 32 + 0.32t$ ft/sec^2.

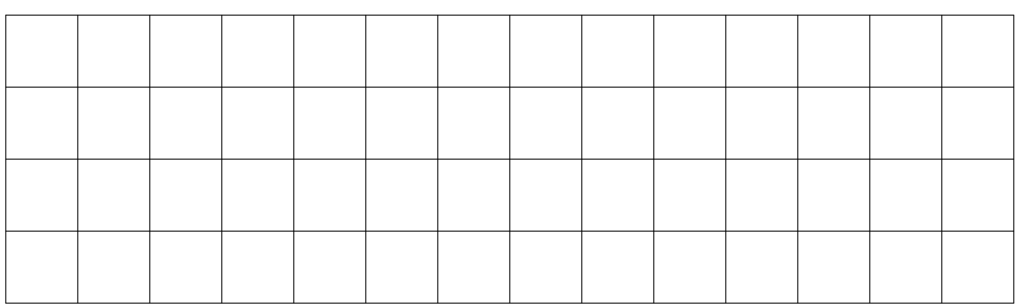
   - (a) How long does it take the rocket to break the sound barrier? (Assume sound travels 1000 ft/sec.)
   - (b) If the rocket burns for 90 seconds, how far has the rocket traveled?

2. Determine whether or not the following are meaningful. Evaluate those that are, and explain what's wrong with those that aren't.

$$\int u^2\,dx \qquad \int u^2 \qquad \int \frac{d\,\text{cabin}}{\text{cabin}} \qquad \int \frac{1}{x} \qquad \int d\theta \qquad \int \frac{e^u}{2x}\,dx$$

3. A graph $y = f(t)$ is shown. Sketch the graph of $F(x) = \displaystyle\int_1^x f(t)\,dt$ in the grid provided, marking the coordinates of all critical points.





4. Find $\dfrac{d}{dx}\displaystyle\int_1^x (1+t^2)^{10}\,dt$. (Hint: Do not evaluate the integral!)

5. Use the substitution $u = e^t$ to transform $\displaystyle\int_0^x e^{e^t}\,dt$. Be sure to handle the limits.

6. Find a formula for $F(x) = \displaystyle\int_0^x 2te^{-t^2}\,dt$, then use your formula to compute $F'(x)$. What happens if you change the lower limit of integration in the definition of F?

7. Without using your notes, find formulas for the given integrals:

$$\int \frac{dx}{a+x}, \qquad \int \frac{dx}{a-x}, \qquad \int \left(\frac{1}{a+x}+\frac{1}{a-x}\right) dx, \qquad \int \left(\frac{1}{a+x}-\frac{1}{a-x}\right) dx.$$

Simplify your answers if possible. For the last two, use algebra to put the integrand over a common denominator. One of the resulting integrals can be done using substitution. Do this, and verify that you get the same answer either way.

8. Evaluate the following integrals. In each case, use a right triangle to simplify the integrand before you integrate.

$$\int_0^x \tan(\arcsin t)\,dt, \qquad \int_0^x \sin(\arctan t)\,dt.$$

9. Use the identity $\cos^2\theta = \frac{1}{2}\big(1+\cos(2\theta)\big)$ to evaluate $\displaystyle\int \cos^2\theta\,d\theta$ and $\displaystyle\int \cos^4\theta\,d\theta$.

10. Use the previous question to compute $\displaystyle\int_0^\pi \cos^2\theta\,d\theta$. Explain your result geometrically.

11. Use substitution and/or integration by parts to compute

$$\int_5^8 x\sqrt{9-x}\,dx \qquad \int \frac{\sin\theta}{\cos^2\theta}\,d\theta \qquad \int x^5\sqrt{2-x^3}\,dx \qquad \int_0^1 2x\arctan x\,dx$$

12. Perform necessary algebra (expansion, polynomial division, completion of the square, partial fractions) on the following, then compute the integrals.

$$\int \frac{x^4+1}{x^2+1}\,dx \qquad \int \frac{x^3-2x^2+x}{x^2-3x+2}\,dx \qquad \int \frac{x+1}{x^2+6x+13}\,dx \qquad \int \frac{2x-4}{(x^2-4x+5)^3}\,dx$$

13. Find the exact area under the first arch of $f(x) = x^2 \sin x$.

14. Use integration by parts to compute $\int v \arcsin(v)\,dv$

15. Evaluate *two* of the following:

$$\int \frac{1}{\sqrt{1+x^2}}\,dx, \qquad \int \frac{x}{\sqrt{1+x^2}}\,dx, \qquad \int \frac{x^2}{\sqrt{1+x^2}}\,dx, \qquad \int \frac{x^3}{\sqrt{1+x^2}}\,dx.$$