

College of the Holy Cross, Spring Semester, 2005
Math 132, Midterm 3 (All Sections)
Wednesday, April 27, 6 PM

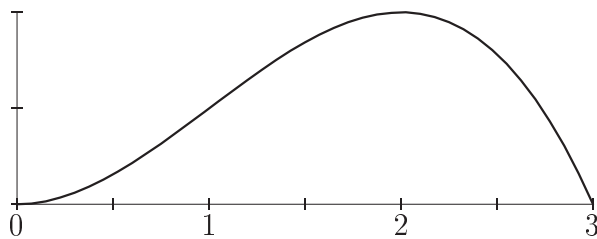
Name: _____

Instructions Please write your answers in the spaces provided, and show work on the test itself. **For possible partial credit, you must show work.** Use the back of the preceding page if you need more space for scratch work.

Please do not write in the space below

Problem	Points/Poss
1	/ 14
2	/ 12
3	/ 12
4	/ 10
5	/ 12
6	/ 14
7	/ 14
8	/ 12
Total	/100

1. [7 points each] Suppose that the length x (in cm) of a certain type of inchworm has density $p(x) = c(3x^2 - x^3)$ for $0 \leq x \leq 3$.



- (a) Find the value of the constant c .

$c =$

- (b) Find the proportion of inchworms with length greater than 1 cm.

Fraction of population longer than 1 cm

2. [6 points each] Suppose

$$P(x) = \frac{4}{\pi} \arctan(x)$$

is the cumulative distribution function for the quantity x , for $0 \leq x \leq 1$.

(a) Find the mean of x .

Mean of x

(b) Find the median of x .

Median of x

3. [6 points each] Use the indicated test to determine whether the given series converges or diverges. You do not need to compute the sum of the series.

(a) $\sum_{n=1}^{\infty} \frac{n^2}{n^3 + 1000000}$; integral test

Converges

Diverges

(b) $\sum_{n=1}^{\infty} \frac{n}{4n^3 + 3n^2 + 5}$; comparison test

Converges

Diverges

-
4. [5 points each] Use the geometric series formula to evaluate (find the sum of) the following:

(a) $5 - \frac{10}{3} + \frac{20}{9} - \frac{40}{27} + \frac{80}{81} - \cdots$

Sum

(b) $\sum_{n=1}^{\infty} \frac{3^n + 7}{4^n}$

Sum

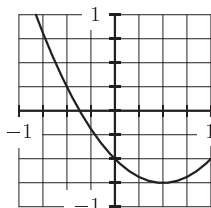
5. Suppose $f(0) = 0.5$, $f'(0) = -1$ and $f''(0) = -2$.

(a) [9 points] Write down the Taylor polynomial of degree 2 for f near $a = 0$.

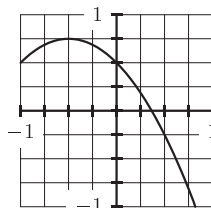
Taylor polynomial:

(b) [3 points] Which (if any) of the following could be the graph of f ?

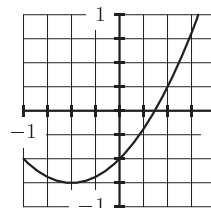
- (i) (ii) (iii) (iv)



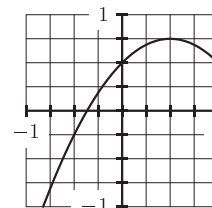
(i)



(ii)



(iii)



(iv)

6. [14 points] Let $f(x) = \sqrt{x}$. Use **the definition** to calculate the Taylor polynomial of degree 3 for f centered at $a = 4$.

7. [7 points each] Both parts refer to the power series $\sum_{n=1}^{\infty} \frac{(x-1)^n}{n \cdot 5^n}$.

(a) Use the ratio test to find the radius of convergence.

Radius:

(b) Investigate the endpoint behavior, and determine the interval of convergence.

Interval of Convergence:

8. (a) [6 points] Use the power series for $\sin x$ near $x = 0$ to find the power series for $\sin(x^2)$ near $x = 0$. Express the series in summation form, and write out the first four nonzero terms.

Summation form:

First four terms:

- (b) [6 points] Use the first four nonzero terms in the result from part (a) to estimate the value of the integral

$$\int_0^1 \sin(x^2) dx.$$

Estimate: