Mathematics 134 – Intensive Calculus for Science 2
Things to Know – Final Exam
April 29, 2002

General Information

- The final exam will be given in our regular class room 8:30am - 11:30am on Tuesday, May 7.
- This will be a *comprehensive exam* – it will cover all of the material from Chapters 5-11 of the text that we have studied this semester. See the detailed list of topics below.
- The final will be similar in format to the three full period midterm exams, but roughly *twice as long*.
- If you are well prepared and work steadily, it should be possible to complete the exam in about 2 hours. But you will have the full-three hour period to work on the exam if you need that much time.
- If there is interest, we can up a daytime or evening review session during study week.

Topics to Be Covered

1) The definite integral, connections with areas, average values, total change.
2) The Fundamental Theorem of Calculus: If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $[a, b]$, then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

3) Antiderivatives graphically and numerically
4) The power, sum, and constant multiple rules for antiderivatives
5) Indefinite integrals by substitution, parts and using the table of integrals
6) Integrals for computing volumes of solids with known cross-sections: $V = \int_a^b A(x)\ dx$, where $A(x)$ is the cross-section area function. Volumes of solids of revolution are a special case of this.
7) Integrals for computing total mass/center of mass of wires and plates of given shapes, given the mass density function
8) Probability density functions of distributions, cumulative distribution function, median, mean.
9) Taylor polynomials and Taylor approximations
10) Taylor series by "shortcut rules" (substitution, algebraic manipulations, term-by-term integration and differentiation) for computing Taylor series from the known series for

$$e^x,\ \sin(x),\ \cos(x),\ (1+x)^p.$$

11) Differential equations
    a) What it means for a function to be a solution of a differential equation

b) Graphical meaning of solutions via slope fields
d) Formulas for solutions via separation of variables
e) Applications to growth/decay problems

Suggestions on How to Study

A) Reread your class notes and work through the examples we did in class – everything on the final will be very close to something we did along the way somewhere!
B) Look over your quizzes and exams, especially any questions you had difficulty with. Read the comments and corrections – they're there to help you figure out what you did wrong and how to solve the problem correctly.
C) Also look at your graded problem sets and the write-ups from the group discussion days.
D) Look over the practice exams for the three full-period midterms.
E) Don't panic! There is a lot of material here, but not all that many key ideas. *Everything on the final will be very close to something we did along the way somewhere!*

The following is the final exam from last year's Intensive Calculus class. Note: In addition to problems like these, I might *also* ask you to approximate a definite integral with a Riemann sum, and/or ask you to compute a center of mass for an object with a given mass density function.

I.
A) (10) Give a precise statement of the Fundamental Theorem of Calculus.
B) (15) The following graph shows $y = f(x)$. Let F be the antiderivative of f with $F(0) = 0$ and F continuous. Sketch the graph $y = F(x)$.

II. Compute each of the following integrals. If you use an entry from the table, say which one.

A) (10) $\int 5x^4 - \sqrt{x} + \frac{3}{x}\ dx$
B) (10) $\int x^3 \sin(x^4 + 1)\ dx$
C) (10) $\int x^2 e^{-5x}\ dx$
D) (10) $\int \frac{1}{4x^2+9}\ dx$
E) (10) $\int \frac{(\ln(x))^4}{x}\ dx$

III. Let R be the region bounded by $y = x^{3/2}$, the x-axis, and $x = 0$, $x = 4$.

A) (12.5) Find the volume of the solid obtained by rotating R about the x-axis.

B) (12.5) Find the volume of the solid obtained by rotating R about the line $y = -4$.

IV. At a particular location in Natick on the Mass Pike, a sensor was set up to measure the passage of traffic. The measurements made were used to derive a probability density function for the quantity x = time gap between successive cars. The results gave the following formula as a good fit for the density:

$$p(x) = \begin{cases} 0 & \text{if } x < 0 \\ .1e^{-.1x} & \text{if } x \geq 0 \end{cases}$$

A) (12.5) Show that p satisfies the usual property for a probability density function: $\int_{-\infty}^{\infty} p(x)\, dx = 1$.

B) (12.5) What was the *median* time gap between successive cars?

V.

A) (10) Using the definition of Taylor polynomials, compute the Taylor polynomial of degree $n = 3$ for $f(x) = xe^x$ at $a = 0$.

B) (10) Use your answer from part A to approximate $f(.1)$, and compare with a calculator value.

VI.

A) (10) Use our "shortcut methods" to find the first four nonzero terms in the Taylor series for $f(x) = \frac{1}{1+x^2}$ at $a = 0$.

B) (10) Use your answer from part A to derive the first four nonzero terms in the Taylor series for $g(x) = \arctan(x)$.

C) (5) Is the graph $y = \frac{\arctan(x)-x}{x^3}$ concave up or concave down at $x = 0$? How can you tell?

VII. According to a simple physiological model, an athletic adult needs to consume food giving 20 calories per day per pound of body weight in order to maintain that body weight. If he or she consumes more or fewer calories, then *his or her body weight will change at a rate proportional to the difference between the number of calories needed to maintain body weight and the number of calories actually consumed.*

A) (10) Say the person's calorie intake is constant at I calories per day. Write the statement in italics above as a differential equation for the body weight W as a function of t (time).

B) (10) Find the general solution of your equation from part A.

C) (10) The constant of proportionality in the relation above has been measured to be $\frac{-1}{3500}$. If a person starts out weighing 160lb and consumes 3000 calories per day, what will his weight be in one year (365 days)?