Mathematics 134 – Intensive Calculus for Science 2
Exam 3 – Things To Know
April 19, 2002

General Information

As announced in the course syllabus, the third full-period midterm exam of the semester will be given in class on Friday, April 26. The format will be similar to that of the midterm exams last semester and Exams 1 and 2 this time. It will cover the material since the last exam (Quizzes 7,8,9), including the following material from Chapters 8,9,10 and 11 of the text:

1) Random variables, probability densities, cumulative distributions, mean, median (Sections 8.6 and 8.7)
2) Geometric series (Section 9.1)
3) Taylor polynomials and series (the basic formula), plus "shortcuts" (substitution, algebraic manipulations, term-by-term integration and differentiation) for computing Taylor series from the known series for

$$e^x,\ \sin(x),\ \cos(x),\ (1+x)^p.$$

(Sections 10.1, 10.3)
4) Differential equations: What it means for a function to be a solution of a differential equation, graphical meaning of solutions via slope fields (Sections 11.1, 11.2)

Important Note: These chapters include a number of sections we did not cover in class. As is only fair, *you are not responsible for any of that material.*

A copy of the table of integrals from our text will be provided.

There will be a review for the exam in class on Thursday, April 25.

Sample Exam

Caution: The following problems indicate *only*

- the kinds of topics that will appear on the exam,
- and the approximate level of difficulty of the questions that will appear.

The actual exam questions *will look different from these.* Some probability questions may require other methods of integration (substitution, parts, the table). The exam itself will be slightly shorter. It is important to understand the methods involved in the solutions to these problems as well as the answers. Do other practice problems besides these ones if you want to be well-prepared.

I. The amount of time t (in seconds) it takes for a car driver to react to an emergency and apply the brakes is a random variable with probability density function $p(t) = \frac{c}{t^3}$ for $1/10 \leq t \leq 5$, and 0 otherwise.

A) What is the value of c? (Use this value in the next parts.)
B) What is the probability that a randomly selected driver will react in some t, $1 \leq t \leq 3/2$ seconds?
C) What is the *mean reaction time* of these drivers?
D) If the car had sensors that could determine when an emergency was occuring and a siren that went off automatically 1 second after the emergency was detected, what fraction of the drivers would already have applied the brakes before the siren went off?
E) Also possible: questions involving the exponential density ce^{-cx} for some c. For these you will need to use substitution to compute probability integrals, and parts to compute the integral for the mean.

II. Only one of the two following infinite series is geometric:

$$\text{series A} : 4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \cdots \qquad \text{series B} : 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

Which one is geometric, and does the geometric series add up to a finite sum? If so, what is the sum? If not, say why not.

III.
A) Compute the 4th degree Taylor polynomial for $f(x) = \ln(1 + x)$ at $a = 0$, *using the definition* of Taylor polynomials.
B) Use your polynomial to approximate the value of $\ln(1.1)$. What is the error in your approximation?

IV.
A) Using our "shortcut" rules, determine the first three nonzero terms in the Taylor series at $a = 0$ of the functions $\cos(x)$ and $(1 - x^2)^{\frac{1}{2}}$.
B) Using the Taylor series, determine which of the two functions in part A is *larger* for small values of x (say $x < .1$). Explain how you can tell.

V. All parts of this question refer to the differential equation

$$\frac{dy}{dx} = y(4 - y)$$

A) Sketch the slope field of this equation, showing the slopes at points on the lines $y = 0, 1, 2, 3, 4, 5, 6$
B) On your slope field, sketch the graph of the solution of the equation with $y(0) = 1$.
C) Also possible: problems like those from 11.2 where the plot of the slope field is given, and you need to plot solutions, or identify the formula for the slope function.

Also see Problem Sets 8,9,10, the last three quizzes, and the Review Problems from the ends of Chapters 8,9,10,11 in the text for other practice problems.