CORRECTION TO: ON WEBS OF MAXIMUM RANK, GEOMETRIAE DEDICATA 31 (1989), 19-35

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December 2, 2002

Abstract. We correct an error in Example (3.4) of [1].

The author would like to thank Prof. V.V. Goldberg for bringing it to his attention that the the result claimed in Example (3.4) of [1] is not true in general.

The error lies in the proof of Lemma (3.5). The assertion that equation (11) implies that

$$Span(p_{12}, p_{13}) \cap Span(p_{14}, p_{15}, p_{16}) \neq \emptyset$$

is not valid. The equation (11) only implies that the p_{1j}, $j = 2, \ldots, 6$ lie in a $\mathbb{P}^4$, not that they lie in a $\mathbb{P}^3$ as claimed. As a result, there is no reason why $Span(p_{12}, p_{13})$ must meet $Span(p_{14}, p_{15}, p_{16})$.

Indeed, there are examples of 6-webs of codimension 2 in 6-dimensional spaces ($W(6,3,2)$ in the notation of the article) that have maximum rank $\pi(6,3,2) = 1$, but that are not almost-Grassmannizable. One way to construct such a web W is to consider 6 families of codimension-2 linear subspaces of $U = \mathbb{R}^6$ or $\mathbb{C}^6$ as follows. Let $u_1, u_2, u_3, v_4, v_5, v_6$ be the coordinates in U, choose an arbitrary invertible 3×3 matrix M and define:

$$\begin{pmatrix} u_4 \\ u_5 \\ u_6 \end{pmatrix} = M \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$$

and

$$\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = -M^{tr} \begin{pmatrix} v_4 \\ v_5 \\ v_6 \end{pmatrix}.$$

The leaves of the ith foliation of W are defined by equations $u_i = const, v_i = const$.

It is easy to check that the following abelian equation holds for every such W:

$$\sum_{i=1}^{6} du_i \wedge dv_i = 0.$$

1991 *Mathematics Subject Classification.* 53A60.

However, in order for W to be almost-Grassmannizable, the 6 web normals $du_i \wedge dv_i$ must all represent lines on the same rational surface scroll (see Theorem 1 of [1]). In this case the relevant scrolls are constructed by taking two $\mathbb{P}^2$'s in general position in $\mathbb{P}^5 = \mathbb{P}T^*(U)$, selecting a conic in each of these planes and an isomorphism from one conic to the other, then joining related pairs of points and taking the union of those lines.

Writing

$$M = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix},$$

the 6 points in the intersection of the $\mathbb{P}^2$ defined by $v_4 = v_5 = v_6 = 0$ and the web normals, i.e. the points with homogenous coordinates

$$[u_1, u_2, u_3] = [1, 0, 0], [0, 1, 0], [0, 0, 1], [a, b, c], [d, e, f], [g, h, i],$$

must all lie on one conic section. That implies

$$\det \begin{pmatrix} ab & ac & bc \\ de & df & ef \\ gh & gi & hi \end{pmatrix} = 0.$$

Hence not every such M yields an almost-Grassmannizable web. Indeed, a necessary and sufficient condition for almost-Grassmannizability of these webs is that $MM^{tr} = D$ for some (invertible) diagonal matrix D.

References

[1] Little, J., *On webs of maximum rank*, Geometriae Dedicata **31** (1989), 19-35.

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