MARK ANDREW FALLER
The Philosophical Use of Mathematical Analysis
(Under the direction of EDWARD HALPER)

This dissertation defends the thesis that Plato's employs methods of philosophical analysis that are akin to and based upon mathematical analysis. It identifies and describes three kinds of ancient mathematical analysis, rectilinear, dioristic, and poristic. It then shows that there are corresponding philosophical modes of analysis in portions of the *Meno* and *Phaedo*. Recognizing Plato's method in these dialogues provides insight into the doctrines Plato advances there and the arguments that support them. It also makes it possible to address three scholarly controversies. First, there is a dispute between F. Cornford and R. Robinson on whether mathematical analysis is a deductive or non-deductive procedure. The dissertation shows that some types are deductive and some are not. Second, there is an issue raised by K. Dorter whether the philosophical method of hypothesis proceeds ultimately to an unconditioned first principle, the good. The dissertation shows how philosophical analysis that is modeled on mathematical poristic could ultimately arrive at an unconditioned principle. Third, there is the question raised by Charles Kahn and others whether the divided line of the *Republic* delineates methods, each of which is applicable to a different type of being. The dissertation shows that, though some methods are most properly applied to particular types of being, Plato applies all the different types of philosophical analysis to all the types of being.

INDEX WORDS: Plato, Philosophical analysis, Geometrical analysis, Mathematical analysis, Method of hypothesis, Geometry, Logic, Syllogism, Diorism, Porism, John Playfair, Francis M. Cornford, Richard Robinson, David Lachterman

PLATO’S PHILOSOPHICAL USE OF MATHEMATICAL ANALYSIS

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DEDICATION

To my father, who wanted this dream so badly for me, and my mother, whose encouragement and sacrifice made it possible for me to see it through.

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TABLE OF CONTENTS

CHAPTER 1

INTRODUCTION

Contemporary philosophy seems to spend much of its time trying to free itself from one of two horns of doubt. The one, skeptical nominalism, a legacy of Ockham and Hume, holds that since all knowledge originates either in reason, and is analytically empty, or in sense experience, and is merely particular, there is no possibility for an objective, universal knowledge of the world. The other pole, the constructive relativism of Berkeley and Nietzsche, maintains that since there are no grounds by which to distinguish absolutely the objective from the subjective, there is no meaningful difference at all. The first gives us the model of a vicious dichotomy. The second demands that we accept an undifferentiable continuum. Within neither model can a robust philosophy of inquiry take place - one whose goal is objective, universal knowledge.

Plato faced these same kind of adversaries two millennia earlier and attempted to construct a method of inquiry which, while steering clear of both horns (Heraclitus, Protagoras), also captured the positive contributions of the many dogmatic philosophies which preceded him (Empedocles, Pythagoras, Parmenides, Anaxagoras). Heraclitus assumes that everything is in flux and that, therefore, to know something as it is at one instant is not to know it at all; and, thus, "not to know something *completely*, is to *completely not know it (Meno)*. Or in other words, the only alternative to absolute knowledge, is absolute ignorance. Protagoras' relativism, on the other hand, was rooted in the Parmenidean impossibility of speaking falsely: "If I can't say *what is not*, then everything which is said, must in some sense *be* (*Euthydemus* 286d*)*." Plato counters by stating his own dichotomy, which is a modification of the Parminidean injunction: "What fully is, is fully knowable, and what in no way is, is altogether unknowable (*Republic 477a*)." The difference between this dichotomy and that of skepticism, is that there is the possibility of a middle ground for both being and knowing. I will argue that Plato has a "degrees of knowing" approach to counter relativism: What "is" even in some degree can be known to that degree in respect of which it "is". The awareness of what we can't know, and the degree and respect to which we cannot know it, can equally be a kind of knowing.

This procedure of "learning" the limits of what we cannot know and why we cannot know it is that of analysis. Analysis is the complete rejection of "Meno's paradox of inquiry" in that "it requires that nothing be so unknown (or unknowable) that it cannot be at least partially assimilated to what is already known (or knowable)[1]." Analysis is the method by which we can systematically "recollect" what we in some way already know.

<u>The Method of Analysis</u>

1.1 Goals of Inquiry

At *Meno* 87a, Socrates proposes to Meno that if they are to investigate whether virtue is teachable or not they must follow the way of the geometers, and proceed by way of hypotheses. Socrates gives a brief description of geometric analysis and suggests that they somehow adapt this method for their own search.

This assertion about following the geometers, along with additional support from ancient mathematical commentators such as Proclus[2], has grown into the dual claims that first, Plato was the inventor of geometric analysis, and second, that the success of his

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[1] This is David Lacterman's description of Cartesian Analysis: *The Ethics of Geometry* (New York, 1989),

philosophy is somehow closely tied to a method of conceptual development which had been appropriated from mathematics.

I will defend the thesis that the method of hypothesis is part of a larger method, most generally known as philosophical analysis. In philosophical analysis, Plato uses hypothesis, along with its complement dialectic, to construct theoretical models of explanation. The way in which he utilizes these tools to develop and interpret conceptual models is entirely parallel to the way in which geometers develop and analyze new constructions within their demonstrations. So I will conclude that philosophical analysis is based on the same model of inquiry as geometrical analysis.

Along the way to proving these claims, I look to accomplish three further ends. First, I would hope that by appreciating some of the practices and procedures of ancient Greek geometry, we might better understand how those same activities influenced Plato's philosophic method. It is certainly not coincidental that at many of the junctures of explaining philosophical method Plato makes reference to specific problems in mathematics (*Meno,* 87a; *Theaetetus,* 148a; *Timaeus,* 32a). In order to accomplish this end I will have to be able to identify those specific structures that can be isolated as mathematical analysis and then turn around and show that these same structures appear in the arguments from the dialogues.

In deciding this first issue I will gain two additional benefits. First, I will then be able to explain how there can be so many distinct positions on the issue of what is analysis. And second I should be able to clarify the methodological question as to

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p 162.

[2] Proclus, *A Commentary on the First Book of Euclid's* *Elements* (Princeton, 1970), p. 165.

whether analysis, as a mathematical heuristic, is an algorithmic procedure or a psychological guide.

The second major end that can be achieved through this study is a deeper recognition as to the nature of mathematical inquiry itself. In this examination we will be working out the details of the logical structure and the directionality of mathematical thinking. Answering some fundamental questions about the nature of mathematical thought will, I hold, shed light on the coordinate question of what philosophical thought is. Mathematics seems both more procedurally rigorous and constructively expansive in its ability to explore, and then demonstrate, new frontiers of understanding. It has been the dream of many philosophers that philosophy might attain either a system of proof, or a heuristic of discovery, from the seemingly successful and progressive paradigms of mathematics.

The third major end is to decide whether and how Plato was able to utilize the procedures of analytic geometry to construct arguments that would defeat the sophists and skeptics. In this effort we must recognize not just the negative consequences of his rebuttals to relativism and skepticism, but see how these same arguments also provided the positive foundations for a robust approach to the possibility of epistemic inquiry.

Examining the question of method with a philosopher like Plato is triply difficult. First, Plato himself seems vague and equivocal when stating exactly what his methods are. The Method of hypothesis and dialectic seem to be the two major methods which he elaborates in the dialogues; however, his description and use of both of these terms are seldom completely compatible in any two, distinct presentations.

Second, as he uses these methods to examine progressively more difficult subject matters, both how the methods are applied and what the methods can aspire to seem to adapt to differing contents. As I will try to show, after explicitly stating to which subjects dialectic and hypothesis are properly applied, Plato will almost immediately apply them to apparently inappropriate subjects.

Finally, neither method, when recognizable, seems ever to be utilized in isolation from the other. As Plato's methods develop in the dialogues, they seem to get progressively more convoluted and intermixed. The main task of this first chapter will be to examine historical and secondary sources on these topics and attempt to come to some reasonable, preliminary 'hypothesis' as to the best way in which to utilize these terms of method.

One problem we will need to anticipate in this examination is the way in which different commentators will emphasize disparate parts of the procedure to certify distinct directions. To demonstrate an upward direction, one author may focus on the *formation* of an hypothesis, while another, to show the downward progression of the method, strictly deals with how consequences are drawn from them. We will need to be extremely sensitive to the complex nature of these procedures so that the question of which way the 'directionality' of the method works does not become needlessly confused.

Before we go very far down the road of exploring Plato's possible methods, I should perhaps say a word about the question of methods in general. In this way we can try to locate Plato's specific approach to method within the context of the available possibilities. There seem to be two poles between which views about methods of discovery have typically been framed.[3] The one holds that methods are an explicit set of rules which are meant to "direct the mind" precisely and rigorously toward the "calculation" of new truths. This approximates closely the attitude toward method that Descartes seems to have espoused. Such *algorithmic* systems would approximate a logic of discovery.

The other pole suggests, instead, that discovery is a 'creative' act. Although the psychological disposition for such activity can be cultivated or acclimatized, it is not absolutely prescribeable. Each of us must find her own way to this creative space, and not all of us are equally capable of attaining it. Method can only be a *heuristic* device to nurture more and better opportunities for "divine" inspiration.

1.2 Interpretive Background

The contemporary debate on the issue of what ancient geometrical analysis is takes its origin from an article written in a 1933 issue of *Mind* by Francis Cornford[4] and the response it elicited from Richard Robinson[5]. The dispute between these two has framed much of the contemporary discussion on the nature of the analytic method. Both authors rely heavily on fragments on the nature of geometrical analysis from *The Treasury of Analysis* of the Alexandrian mathematician, Pappus (320 AD). Pappus reports that Plato "discovered" the analytic method.

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[3] I will restrict my comments to methods of 'discovery' for the present time, ignoring the possibilities of method as 'construction'. The 'platonic' is universally understood as signifying that perspective which assumes the reality of that which one can know. In mathematics and logic, Platonists believe in the real 'existence' of the principles and laws. And what already 'is' can only be discovered, not constructed. Although my analysis will eventually challenge this simplistic framing of Plato's view, it will be adequate for our present examination.

[4]Francis Cornford, "Mathematics and Dialectic in the Republic VI-VII.(I.), *Mind* 41 (1932): 37-52.

[5]Richard Robinson, "Analysis in Greek Geometry", *Mind* 45: (1936) 464-73.

I will attempt to utilize the demarcations of this dispute as to the nature of the analytic method to set out the guidelines for exploring Plato's philosophic method. In particular I will examine the way in which the disagreement over whether this method was deductive or non-deductive in orientation helps determine our sense of "directionality" within philosophical analysis. Is analysis "upward" (towards the universal) or "downward" (from the universal).

Cornford's article was intended to reverse what was an accumulating consensus by modern historians of mathematics[6]that ancient Greek geometrical analysis just was a primitive form of modern algebraic analysis. This view held that analysis was a system of biconditional, and therefore reversible inferences, following from an initial assumption. Since analysis was completely reversible, once the analytic conclusion was reached, it could in turn be used as the premise for a deductive syllogism proving the assumed hypothesis. In other words, analysis was merely a form of reversible deduction (A = B = C =…).

Cornford betrays both a positive and negative motivation behind his effort to reinterpret the phenomenon of analysis. The negative axis is easier to identify: Cornford believed that many of his contemporaries had projected a "modern" view onto the geometrical analysis of the ancients. They were interpreting ancient structures of thinking through the lens of modern logical theory. In particular, mathematical commentators[7] had misinterpreted an important phrase in Pappus' description of ancient geometrical analysis. They had translated the words, 'δ4• τäν ©ξ-Η •κολοÛ2ων'[8] as a passing from the hypothesis "through its successive consequences" rather than as Cornford would as "through the *sequent steps*". An ordinal connection had been changed

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[6] Heath, Sir Thomas, *A History of Greek Mathematics* (New York, 1981).
[7] Robinson, p. 469, is presenting Cornford's argument.
[8] Ibid., p. 468.

into a logical one. The modern commentators had transformed analysis into simple deduction.

Cornford seems rightly disturbed at the lack of precision in the use of logical terminology by these mathematical commentators. Deduction simply can't occur in two directions with the same propositions: " You cannot follow the same series of steps first one way, then the opposite way, and arrive at logical *consequences* in both directions."[9] Analysis and synthesis must surely be distinguishable through some structural or directional principle. Robinson identifies this maxim as Cornford's "a priori" principle, and while conceding that it has a certain theoretical validity, he denies that it holds in the practice of logic[10].

Contemporary logicians understand the "deductive syllogism" as equivalent to an identity or "equation". All logical movement derives from substituting terms with equivalent meaning into such "analytic"(in the modern sense) identities. Robinson actually uses equations to demonstrate his point[11]: "The following three propositions seem to form a series that will give logical consequences in either direction: (1) $3x = 4y$, (2) $3x + y = 5y$, (3) $3x + 2y = 6y$."[12] As such there is no problem of the convertibility of such extended identities, since all mathematically derived deductive entailments are assumed to be "biconditional". Equations are the paradigm of the conversion relationship, and by virtue of the expression of geometrical relations in equational form modern scholars no longer attempt to differentiate the logical schemata of geometry from that of algebraic analysis.

For the ancients, however, syllogisms are strictly directional relationships, even those associated with mathematical demonstrations. Convertibility is a serious concern for early logicians in general, and, as we shall see, Plato in particular. If the propositions

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[9]Cornford, p.47n.
[10]Robinson, pp. 468-69.
[11]Ibid., p. 469.
[12]Ibid.

of geometry are more likely to convert, there must be some reason or account for this special disposition.

We should recognize at this point there are really three positions at issue here. One holds that analysis is not deductive at all (Cornford). A second holds that analysis is deductive just because all of the propositions are convertible (Robinson). But there is a third possibility. Analysis could be convertible with deduction without *all* of its propositions being convertible (traditional syllogistic).

This further issue of logical structure introduces Cornford's positive impetus for the essay. He had, I believe, a rare insight into the way in which mathematicians in general and Plato in particular are conscious of how they actually "discover" the ordering principles of new theorems: they work backwards. This working backwards is, in most cases, not a mere deduction or a calculation, but rather a kind of *uncovering* which relies on a combination of logical thoroughness, intuitive insight, and lucky guesses. Mathematical philosophers have long championed the priority of the analytic over synthetic methods with regard to their “fertility” for nurturing developing thought[13].

Cornford believed that he had identified an axis of distinction between *noesis* and *dianoia* within Plato's methods of dialectic. *Dianoia* was exemplified by the systematic passage of logical propositions through well defined concepts. For Cornford, analysis, an activity of *nous*, involved an intuitive leap of "divine madness" which could not be fully transparent to the rules of deduction: “It is certain that in his account of the dialectical ascent Plato is describing the upward movement of thought which has been illustrated from geometrical analysis.”[14]

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[13]Fermat almost never provided synthetic demonstrations of his discoveries, believing them superfluous to the reasoning of analysis (Mahoney, *The Mathematical Career of Pierre de Fermat* [Princeton, 1994], p. 85). Descartes held that philosophers should throw off their “syllogistic fetters” (*Rules for the Direction of the Mind,* Rule XII, in *The Philosophical Works of Descartes*, I, trans. Elizabeth Haldane and G.R.T. Ross (Cambridge: Cambridge University Press).
[14]Cornford, p. 47.

As such, it is misleading to identify the sequence of steps with which the mathematician works back from the conclusion towards the necessary and sufficient premises as a deduction. A deduction is a "forward" driven process that is traditionally taken to work from the more universal to the more particular.

Geometrical analysis is rather more closely associated with what Aristotle denotes as ‘deliberation’: "which proposes an end to be achieved by action and then works backwards along the chain of means to that end, till it reaches, as a first link in the chain, an action that can be at once performed."[15] Aristotle specifically compares this process with the analysis of a mathematical diagram: “For a deliberator would seem to inquire and analyze in the way described, as though analyzing a diagram…e.g. in mathematics [inquiry] is deliberation (*Nic. Ethics*,1112b20).”[16] This seems to be a good rendering of the kind of procedure that Plato works through with the slave boy.

Robinson's significant insight into this debate is that there appear to be many kinds of mathematical problems that more closely fit the kind of activity described by that mathematical analysis which Heath and others attribute to Pappus. Pappus is a key figure in this debate since he is one of the few surviving sources of discussion on the method of analysis. In the single, comprehensive statement extant on the nature of ancient analysis, Pappus asserts:

> Now *analysis* is a method of taking that which is sought as though it were admitted and passing from it through its consequences [δ4• τäν ©ξ-Η •κολοÛ2ων] in order to something which is admitted as a result of synthesis; for in analysis we suppose that which is sought to be already done, and we inquire what it is from which this comes about, and again what is the antecedent cause of the latter, and so on until, by retracing our steps we light upon something already known or ranking as a first principle; and such a method we call analysis, as being a reverse solution.
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> But in synthesis, proceeding in the opposite way, we suppose to be already done that which was last reached in the analysis, and arranging in their natural

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[15]Ibid., p. 44.
[16] Aristotle, *Nicomachean Ethics*, tr. Terence Irwin (Indianapolis, 1985).

> order as consequents what were formerly antecedents and linking them one with another, we finally arrive at the construction of what was sought; and this we call synthesis.[17]

Pappus also goes on to recognize two distinct sorts of analysis, as distinguished by the kinds of demonstration sought:

> Now analysis is of two kinds, one, whose object is to seek the truth, being called *theoretical*, and the other, whose object is to find something set for finding being called *problematical*. In the theoretical kind we suppose the subject of the inquiry to exist and to be true and then we pass through its consequences [δ4• τäν ©ξ-Η •κολοÛ2ων]
> in order as though they also were true and established by our hypothesis, to something which is admitted; then, if that which is admitted be true, that which is sought will also be true, and the proof will be the reverse of the analysis, but if we come upon something admitted to be false, that which is sought will also be false. In the problematical kind we suppose that which is set as already known, and then we pass through its consequences in order, as though they were true, up to something admitted, then, if what is admitted be possible and can be done, that is, if it be what the mathematicians call *given*, what was originally set will also be possible, and the proof will again be the reverse of the analysis, but if we come upon something admitted to be impossible, the problem will also be impossible.[18]

Yet Pappus' own comments and examples are not so clearly elaborated as finally to decide the issue. His examples and part of his commentary seem to confirm Robinson's claim that analysis emphasizes the reversible nature of mathematical deduction (“a downward movement from an initial assumption”). Other comments, however, seem to point more directly to the kind of activity recognized by Proclus and Cornford (“an upward movement to prior assumptions”). This ambiguity has led Gulley to assert that Pappus is referring to two distinct models of analysis.[19]

Robinson criticizes Cornford's interpretation for not closely following certain key examples of Pappus, Archimedes and Euclid, where something more akin to modern

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[17]Pappus, “Treasury of Analysis”, *Greek Mathematical Works, II*, ed., Ivor Thomas (Cambridge, MA, 1993), pp. 597-98. Thomas follows Robinson’s interpretation of the controversial Greek phrase.
[18]Ibid., p. 599.
[19]Norman Gulley, "Greek Geometrical Analysis", *Phronesis*, 3 (1958): 13.

analysis seems to be invoked. Robinson also relies on the testimony and practice of modern mathematical analysis. Because *algebraic* analysis (based on equations) is universally convertible, and logical deduction has since been subsumed under algebraic - set theoretical rules, convertibility and deduction must therefore be synonymous. Since Pappus and the later geometers seem to assume a kind of convertibility in their practices, they must be asserting that analysis is deductive.

There are two difficulties with this approach. The first, already broached by Cornford, is that even when analytic procedures are convertible with synthesis, this does not reduce them *simpliciter* to deduction. Deduction is traditionally considered a directional process, moving from logically prior premises downward towards a logically posterior conclusion. Convertibility, on the other hand, is a lateral process by which equivalent expressions are exchanged for each other. We should at least begin with a presumption that ancient geometrical analysis is a distinct phenomenon from modern algebraic analysis.

The other difficulty involves Robinson's convenient blindness to his own insight when it no longer serves his ends. He casually assumes that "almost all" geometrical problems are convertible.[20] Despite his sensitivity to those kinds of problems which seem to defy Cornford's exemplification of analysis, Robinson does not consider other problems more amenable to Cornford's interpretation. Patrick Byrne points out that the example which Robinson uses is pointedly “arithmetic” rather than geometric[21]. Geometry problems tend to be less universally convertible than those of arithmetic, as we will see later. Are there two kinds of analysis, deductive and non-deductive? Is there some family of problems which could systematically determine convertibility?

Robinson admits that some of the language in Pappus seems to support Cornford’s interpretation: “ In analysis we assume that which is sought...and inquire what

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[20] Robinson, p. 469.
[21] Patrick Byrne, *Analysis and Science in Aristotle* (Albany, 1997), p. 7.

it is *from which* this results."[22] If analysis were simply deduction we would expect him to say, "what results from this". To counter this Robinson points out that for Cornford's interpretation to be correct, Pappus would have to be accounted for making a logical mistake. Pappus claims: " if we come upon something admittedly false, that which is sought will also be false."[23] Robinson points out that this statement is true for a deduction but not for the kind of "working back" process of Cornford. As Aristotle points out in the *Posterior Analytics*, " If it were impossible to prove truth from falsehood, it would be easy to make an analysis; for they [i.e. conclusion and premise] would convert from necessity (*Posterior Analytics*, I,12 78a6-10)."

The problem with this criticism is that Robinson is assuming that ancient logic is using the same kind of "material" inference that we moderns use. In such an inference, if any member of the premise set is false, the inference is automatically 'valid'. This is a dangerous assumption to make. In the strict implication which Plato is using, even though false premises may lead to true conclusions, it doesn't thereby *guarantee* the validity of the resultant syllogism. On the contrary, when false premises support a true conclusion, the inference, as the connective necessity between the premises and conclusion, itself remains false. This reference might correspond to the sense in which Pappus is referring to the "obtaining of a false analysis."

One of the elements which makes this discussion so intriguing is that it does not rely on any substantive obfuscation of the opposing positions. It seems that both sides of the issue have non-controversially identified the other's stance. I don't believe either Cornford or Robinson would claim that the other has misrepresented his case.

What is equally interesting is that the source material from which both sides derive their arguments seems equally uncontroversial. Both sides can find support for their positions in the clear but equivocal passage from Pappus.

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[22]Robinson, p. 470.
[23]Ibid., p. 467.

So to decide this contest we need not reconstruct the oppositions, nor negotiate embattled translations. We need rather to be able to determine how Pappus or Plato could have consistently maintained a definition of the analytic method which supports two such radically opposed interpretations.

At this point I will examine the issue of analysis from the perspective of four commentators who have followed up on this initial discourse by Robinson and Cornford. Although I do not believe that any of these authors have finally resolved the issue of "what is geometrical analysis", by examining different aspects of the problem these four commentators will enable us to look more deeply into the full complex of issues that make analysis so difficult to understand.

After this preliminary investigation of the nature of analysis is completed, I will then attempt to examine this procedure in relationship to the more generally recognized methods of Plato's philosophical inquiry - dialectic and hypothesis. In this second, more comprehensive examination I will develop three axes or issues through which different authors have interpreted Plato's method. I will then proceed to utilize these three issues as the organizational structure for elaborating Plato's methods throughout the rest of the dissertation.

Jaakko Hintikka and Unto Remes, in their work *The Method of Analysis*[24], begin by taking apart the steps which are involved in geometrical theorems and problems. By looking at the contribution of each step toward the analysis and resolution of the problem, they are able to determine that the traditional distinctions made between problematic and theoretical analysis are for the most part superficial. Both kinds of analysis basically involve the same types of reasoning, although it takes place in a slightly different ordering.

[24] Jaakko Hintikka and Unto Remes, *The Method of Analysis* (Boston, 1974).

What is essential to all of these procedures, is that they all necessarily involve constructions as a complement to the analysis. For Hintikka, this invariable fact signifies the convergence of three separate but substantial insights into the nature of geometrical thinking. First is Aristotle's insight that analysis is like deliberation in that it is a goal directed thinking about the way to bring about certain ends. In the case of geometry, those ends are the bringing about of certain relationships between geometrical figures.

Hintikka and Remes want to stress that the problem with the traditional way of thinking about analysis is that it has obscured the focus of what analysis is 'of '. Analysis is not 'of propositions' but rather 'of figures'. For this reason the issue of directionality is misconceived. It is not a determination of 'direction' which accounts for the increase of knowledge within the deductive phase of the analytical procedure. It is the taking apart of a figure which establishes the new premises within the proof.

The key insight in this interpretation of the analytic procedure is that the "auxiliary construction", which takes place subsequent to the initial laying out of the problem, imports additional assumptions into the given relationships. Because the relationships are between figures, and not just propositions, the additional constructions are a development of the original "givens".

This brings to light Hintikka's and Reme's second insight: Mathematical thinking can be both deductive and ampliative (synthetic in the modern sense) at the same time. Auxiliary constructions within an analysis supply supplementary premises which allow deduction to "move upward" in a sense.

The point which Hintikka wants to stress here, is that the direction is "upward" in terms of increasing the information available as the analysis proceeds, but "downward" in terms of proceeding by a deductive necessity rather than intuition or divination. In the midst of a synthetic demonstration, the auxiliary construction 'amplifies' the premise set so that the deduction seems to produce an increase in knowledge. It is the influx of new information from the construction which allows this dual procedure to take place.

Michael Mahoney in an article, "Another Look at Greek Geometrical Analysis,"[25] suggests some other possible paths for the interpretation of analysis. Utilizing his extensive work in the historical development of analysis in the modern era,[26] Mahoney systematically develops an understanding of the tools of mathematical analysis and demonstrates how they may have had possible utility to the thinking of the ancient philosophers. He takes the stance that analysis was less a specific method than a body of techniques utilized to solve problems. Like Hintikka and Remes he believes that there was only a single form of analysis, problematic, but unlike Hintikka he is convinced that it was merely a set of heuristic devices rather than an algorithmic procedure.

Although relying on the work of many of the same historians of mathematics as Robinson, Mahoney holds to the thesis that there were non-reversible forms of analysis. To this end he carefully examines the structure of those activities which made up analysis, in particular the procedures of “diorismos” and “porismos”. In these he believes he has located a unique sequence of problem solving techniques, which when used in coordination, can elaborate the full set of conditions for the analysis of any problem. Unfortunately, his presentation is more historical than philosophical. He does not seem to care to go the further distance of interpreting the epistemological consequences of his historical examination. He has recognized an extensive framework of distinct tools within the ancient geometer’s practice, but he hesitates to elaborate on the exact significance that these tools could have had for the conceptual development of ancient models of knowing.

Mahoney’s position is interesting for its singular focus on the practice of the geometers to locate exactly what analysis was. Mahoney sees geometrical analysis as something much broader than the specific logical problems cited by Pappus. He instead

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[25] Michael Mahoney, "Another Look at Greek Geometrical Analysis," *Archives for the History of Exact Sciences* 5 (1968): 319-48.
[26]Mahoney, *Fermat.*

goes deeply into the works of the geometers themselves to identify the multiple tools which were normally referred to as analysis.

After discussing the many different techniques of Euclid, Archimedes, and Apollonius, Mahoney suggests that analysis just was this collection of diverse and powerful tools which the ancients used to frame and solve problems. He claims that the debate about directionality in analysis seems to have been introduced by the philosophers rather than by the practicing mathematicians. As such the directionality of analysis is not a mathematical issue and has little to do with the way the mathematicians speak about analysis. This issue is a confusion brought upon mathematics by philosophers interested in a completely different problem.

While there is certainly much to be gained from the study which Mahoney makes of the practices of ancient mathematicians, he can by no means completely dismiss the issue of directionality with regards to analysis. While it is no doubt true that this issue was of more concern to the philosophers of mathematics than the mathematicians themselves, to suggest that the work of these two groups can be cleanly separated in studying ancient mathematical practice is perhaps naive.

Hintikka and Mahoney, much like Cornford and Robinson, want to simplify the analytic method into a single, clear understanding of a particular aspect of what Pappus seems to be presenting. Although both further the development of possible elaborations of analytic technique, in the end they are equally liable to the same kinds of reductive attitudes which make resolving this complex dilemma more troublesome.

Norman Gulley finds the attempt to make the two descriptions of analysis completely consistent with each other the "weakness of each interpretation."[27] He instead concedes to Robinson that examples from the work of Archimedes and Pappus seem to emphasize the convertibility of each step of the analysis. But he is unwilling to discount

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[27]Gulley, p. 2.

the clear implication of the Pappus' assertion that analysis is a backwards resolution from an original assumption. His alternative is just to accept that there are two forms of analysis referred to by Pappus and to look for confirmation for the non-deductive variety in other historical sources.

And there are a substantial number to be found. He begins with Aristotle, who first utilized the term *analusis* as a philosophical term. Aristotle points to the term's application to such relations as of a whole into its parts, of a compound into its elements and of the complex into the simple.[28] Its logical usage is contrasted as an upward movement with the downward movement of synthesis.

Gulley cites both Albinus and Diogenes Laertius to show that the method of geometrical analysis had a close kinship with that of philosophical analysis. Both philosophers trace the origin of the analytic method to Plato's explaining the technique to his student Leodamas. It is not unreasonable to propose that philosophical and geometrical forms of analysis had a reciprocally beneficial influence in this environment. Yet in all of the many references cited (these include Aristotle, Albinous, Laertus, Proclus, Alexander, Ammonius, Themistius, and Philoponus) Gulley can nowhere find "any account of analysis corresponding to Pappus' account of it as deductive."[29]

Kenneth Sayre, in *Plato's Analytic Method*, attempts to give a comprehensive account of analysis using only Plato's presentation in the dialogues.[30] This approach has certain attractive features. First, it begins to tie together references to geometrical analysis with those of philosophical analysis. While Plato tells us that the two are related, it is still helpful to see how the elaboration of analysis in the dialogues can be compared to the structures enumerated in geometrical problems. In particular, Sayre claims that the analytic method just is the method of hypothesis and that this method,

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[28]Ibid., p. 4.
[29]Ibid., p. 9.
[30]Kenneth Sayre, *Plato's Analytic Method* (Chicago, 1969).

rather than having been left aside for the more advanced techne of dialectic, remains Plato's main philosophical tool throughout the development of the dialogues.

Sayre does a thorough job of showing how the method of hypothesis is both utilized and evolves in both the *Theaetetus* and the *Sophist*. In the process of this exposition, he examines the directionality of analysis in terms of the criteria of necessary and sufficient conditionality. This framework makes discussion of the directional issue within the analytic method easier to clarify. 'Downward' just is from the sufficient condition to the necessary (synthesis), and 'upward' is the reverse (analysis).

Sayre begins by recognizing that the Greek word *aitia*, has two distinct but related usages. It can mean both the 'reason' for or 'cause' of some phenomenon. Sayre determines that Plato's epistemological usage of *aitia* as reason is derivative from and dependent on his ontological use of *aitia* as cause. Rather than interpreting the language of "connectiveness" as merely logical, Sayre makes the reasonable assessment that Plato is referring to "real" connections, from which the logical relations then derive. He distinguishes necessary and sufficient connections as different levels of causal relatedness and points to the convertible relation of necessary and sufficient conditions as the causal relation most proper. By founding the logical connectiveness of 'reasons' on the more robust bonding of causality Sayre is able to take apart more carefully the distinct uses of Plato's imagery of ontic connections.

Sayre makes the case that the frustrated searches for the sophist, in the dialogue of that name, were due to an intentional conflation of the necessary with the sufficient conditions for the definition. After starting off with what Sayre takes to be a good definition of the angler, as including both the necessary and sufficient conditions for the techne, the Stranger goes off on a rather strange fishing trip: "In at least one important respect just reviewed, however, the first five definitions of the sophist which immediately ensue (from 221d to 226a) do not follow this pattern at all. None of them gives the necessary conditions for being a sophist. This is indicated by the fact itself that five competing definitions of the sophist are given. Although each of these is sufficient, in the sense that there are people called ‘sophists who meet each description, none mentions features which, either separately or in combination, are necessary to being a sophist.[31]

Sayre’s solution of this dilemma is just to put all of the errant definitions together and somehow arrive at those conditions that are both necessary and sufficient. In this procedure, Sayre seems to lay much faith in the dialectical laying out of the full range of possible conditions in discovering that unique arrangement which is the true cause or definition. But there are a few points that Sayre seems to be ignoring. First, how exactly, on his account, was Plato able to determine from his divisions which qualities of the sophist were the "necessary" ones? Sayre maintains that there is a hierarchy of divisions that is based upon a hierarchy of necessary qualities of the sophist, but he does not explain how to determine which qualities were more necessary. Also, how is it that we make the original division between productive and acquisitive arts in the first place? For the angler, this division seems somewhat appropriate. For the sophist it does not appear so obvious.

More importantly, Sayre seems to ignore the most significant distinction between the Stranger's two kinds of hunts. In the angler example, the activity of the angler is never in question. We know exactly what the angler does from the very beginning. So the division has a clear and distinct sense of that guiding function within which all the divisions of genera unfold. For the sophist it is very different. While we are familiar with the varied assortment of comportments of this creature, we are never clearly in view of that function which seems to guide and direct all the others. And it would seem that dialectic is in a very poor position to make that distinction. Sayre’s ontic critique of analysis is able to accomplish two important tasks for our inquiry. First, by exploring the more precise elements of causal conditionality, he is able to compare the issues of

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[31]Sayre, p. 144.

directionality and convertibility in a way which keeps them distinct, while showing how they can under certain circumstances interact. Some causal relations are merely directional, others more properly are both necessary and sufficient.

Second, by tying analysis back to the method of hypothesis, we can further develop the many examples which Plato both uses and talks about in the dialogues to better frame the nature of this procedure from its properties and functions. It is clear that we must be sensitive to the failings of those hypothetical explorations in the *Theaetetus* and the *Sophist* if we are to work out the difficulties behind this method.

While Sayre's approach helps to develop the problem of understanding Plato's analysis to a much more complex level than much of the previous discussion, it is not without problems. By not including any references to geometrical analysis, it is not at all clear how the references to causality are to influence directionality and convertibility within geometrical expositions. It seems clear that Robinson is correct in recognizing among the ancient geometers at least some reversible or deductive-like practices in analysis. Sayre's critique, while clarifying and grounding that procedure of discovery which Cornford asserts is analysis, completely ignores these alternatives (Robinson's).

Even at the causal level, Sayre is not perfectly clear on how it is that an understanding of directional causality can be "transformed" into one which is more properly biconditional. It seems not enough merely to have a field of conditions laid out before one, to be able to determine which subset of them are necessary and sufficient. It is this step of *recognizing* these differing conditions *as* necessary or sufficient, which represents the critical mystery in the movement of analysis.

Sayre has not recognized why Plato has offered the distinction between acquisitive and productive arts as key to solving this problem. To determine which are necessary conditions and which are sufficient conditions, one must first have knowledge of the "good" of the techne. Determining this good lies in the more dangerous and confusing realm of production.

There is one interesting *corollary* or by-product turned up by Sayre's inquiry. In claiming that causality proper is biconditional, Sayre has forged a link between the issues of directionality and method which has direct consonances in the dialogues. The method of hypothesis, or analysis, is the backward movement from necessary conditions to those which are sufficient. This corresponds well to the upward way of Pappus. At the point at which the necessary and sufficient conditions are fully elaborated, and the biconditionality of the causal definition is laid out, the logic becomes fully convertible, and the 'downward' way of synthetic demonstration can follow.

What the expositions of Gulley and Sayre have introduced is the need to survey the wider field of Plato's philosophical method if we are to be able to isolate and describe that part of his method which we can more properly identify as analysis. This broader scope, however, brings with it some new problems. It will now be necessary to compare and distinguish our rough sketch of the analytic method with Plato's more commonly recognized forms of inquiry - dialectic and the method of hypothesis.

1.3 Hypothesis and Dialectic

There is little consensus on exactly what the method of hypothesis or dialectic is or how either of them are utilized in the dialogues. The debate seems to divide into three incompatible positions. The first holds that the method of hypothesis was developed in the early middle dialogues, and then abandoned for the more useful dialectic. The evidence offered for this view is the way in which Socrates seems to denigrate the method of hypothesis in the *Republic*, and the fact that there is little direct reference to the method after that dialogue. While the term hypothesis is utilized in the *Parmenides*, Plato seems more clearly to be using dialectical method.

A second view holds that the method of hypothesis is merely a development within and a form of dialectic itself. Commentators like Robinson seem to hold this view from the fact that hypothesis is often used in conjunction with *elenchus* in the early-to-middle dialogues, and *elenchus* is recognized as a form of dialectic.

The third view is a development of the second. Some scholars, taking their cue from the Divided Line, have attempted to discern the difference between dialectic and hypothesis as one of a "direction" of method. While the dialectical method climbs toward the one, the hypothetical method develops consequences of an hypothesis which are directed towards the many.

I will attempt to clarify these differences by examining three separate frameworks from which the nature of hypothesis may be compared and distinguished from that of dialectic. Each of these problematics or frameworks of examination will attempt both to distinguish hypothesis from dialectic, as well as to establish the structure and function of each. The first framework attempts to distinguish dialectic from hypothesis on the basis of logical structure. Parallel to the discussions on the nature of analysis, this axis of examination will seek to classify the two methods in terms of their deductive and non-deductive structural connections.

A second framework used to distinguish the two methods will be that of directionality. This framework will focus on the "teleology" or functional directionality of the methods. To what end and with what goals does the method operate? This issue of *functional* directionality differs from the previous issue, for the latter concerns *structural* directionality. That is, wheras the first framework concerns the direction of movement between premises, the second concerns the end to which those premises ultimately lead.

A third framework will be to understand what the proper subject matter of each method might be. The Divided Line seems to correlate the faculties and methods of knowledge through the possibility and status of their ideal objects. This framework would seem to give a clear and distinct reference for what the appropriate object of each method should be.

1.3.1 First Framework

The issue of the logical structuring of the method of hypothesis is best taken up in Plato's two initial presentations of it in the *Meno* and the *Phaedo*. In the *Meno*, Plato points towards the origination of the hypothetical method in mathematics. He invokes this method of the geometers to illustrate how it is that true opinion can be developed or "tethered" into some kind of knowledge. He proposes to explore whether virtue is knowledge by exploring whether or not it is teachable, on the hypothesis that if it is the one, then it must by consequence be the other. The warrant for this move lies in the mathematical 'gimmick' of assuming as given that which one wishes to prove and then proceeding to see "what follows" from such an assumption:

> Soc. Had I the command of you as well as of myself, Meno, I would not have inquired whether virtue is given by instruction or not, until we had first ascertained "what it is." But as you think only of controlling me who am your slave, and never of controlling
> yourself,-such being your notion of freedom, I must yield to you, for you are irresistible. And therefore I have now to inquire into the qualities of a thing of which I do not as yet know the nature. At any rate, will you condescend a little, and allow the question "Whether virtue is given by instruction, or in any other way," to be argued upon hypothesis? As the geometrician, when he is asked whether a certain triangle is capable being inscribed in a certain circle, will reply: "I cannot tell you as yet; but I will offer a
> hypothesis which may assist us in forming a conclusion: If the figure be such that when you have produced a given side of it, the given area of the triangle falls short by an area corresponding to the part produced, then one consequence follows, and if this is impossible then some other; and therefore I wish to assume a hypothesis before
> I tell you whether this triangle is capable of being inscribed in the circle":-that is a geometrical hypothesis. And we too, as we know not the nature and -qualities of virtue, must ask, whether virtue is or not taught, under a hypothesis: as thus, if virtue is of such
> a class of mental goods, will it be taught or not? Let the first hypothesis be-that virtue is or is not knowledge,-in that case will it be taught or not? or, as we were just now saying, remembered"? For there is no use in disputing about the name. But is virtue taught or
> not? or rather, does not everyone see that knowledge alone is taught?
>
> Men. I agree.

> Soc. Then if virtue is knowledge, virtue will be taught?
> Men. Certainly (*Meno,* 86d-87c[32]).

Socrates immediately proceeds to demonstrate first, that virtue is a kind of knowledge, since only knowledge is good (86e-87b), and then, that it can't be knowledge, since it isn't teachable (87b-89c).

It is interesting to note here that Plato's choice of words point us toward what seems to be a "deductive" procedure: What *follows* from the hypothesis is what is entailed by it. From this passage it appears, at first glance, that the method of hypothesis, like that of analysis, is just a process whereby an inquiry may be transformed into a deductive chain of implications.

The exposition of hypothetical method in the *Phaedo* is both more extensive and complex than that proposed in the *Meno.* Socrates approaches the method in the telling of his 'second sailing' in understanding the nature of causes:

> After this, he said, when I had wearied of investigating things, I thought that I must be careful to avoid the experience of those who watch an eclipse of the sun, for some of them ruin their eyes unless they watch its reflection in water or some such material. A similar thought crossed my mind, and I feared that my soul would be altogether blinded if I looked at things with my eyes and tried to grasp them with each of my senses. So I thought I must take refuge in discussions and investigate the truth of things by means of words. However, perhaps this analogy is inadequate, for I certainly do not admit that one who investigates things by means of words is dealing with images any more than one who looks at facts. However, I started in this manner: taking as my hypothesis in each case the theory that seemed to me the most compelling, I would consider as true, about cause and everything else, whatever agreed with this, and as untrue whatever did not so agree. But I want to put my meaning more clearly for I do not think that you understand me now (*Phaedo,* 99d-100a[33]).

Socrates continues his tale by giving a brief introduction to the theory of forms, after which he finishes his presentation of the hypothetical method:

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[32] Here and throughout this dissertation I will be using G. M. A. Grube's translation of Plato's *Meno* in *Five Dialogues*, (Indianapolis, 1981).
[33] Here and throughout this dissertation I will be using Grube translation's of Plato's *Phaedo* in *Five Dialogues*.

> Then would you not avoid saying that when one is added to one it is the addition and when it is divided it is the division that is the cause of two. And you would loudly exclaim that you do not know how else each thing can come to be except by sharing in the particular reality in which it shares, and in these cases you do not know of any other cause of becoming two except by sharing in Twoness, and that the things that are to be two must share in this, as that which is to be one must share in Oneness, and you would dismiss these additions and divisions and other such subtleties, and leave them to those wiser than yourself to answer. But you, afraid, as they say, of your our own shadow and your inexperience, would cling to the safety of your own hypothesis itself, you would ignore him and would not answer until you had examined whether the consequences that follow from it agree with (*sumphonein)* one another or contradict one another. And when you must give an account of your hypothesis itself, you will proceed in the same way: you will assume another hypothesis, the one which seems to you best of the higher ones until you come to something acceptable, but you will not jumble the two as the debaters do by discussing the hypothesis and its consequences at the same time, if you wish to discover any truth. This they do not discuss at all nor give any thought to, but their wisdom enables them to mix everything up and yet to be pleased with themselves, but if you are a philosopher I think you will do as I say.
>
> What you say is very true, said Simmias and Cebes together (*Phaedo*, 101b-102a).

This second version of the hypothetical method both extends and conflicts with the version in the *Meno*. We are again informed that the method asserts that there is a movement from one hypothesis to another which is in some sense "simpler." There is again in this passage, like the one in the *Meno,* a reference to consequences, which suggests a kind of deduction of truth. The reference to "higher" however, seems to suggest that the movement is not in the deductive direction (downward) but rather backwards towards prior premises.

There is also the added imagery of "accordance" or "agreement" in this presentation. The Greek term *sumphonein* has significations that go beyond the association with logical connection. Besides the reference just cited, there is the earlier assertion: "taking as my hypothesis in each case the theory that seemed to me the most compelling, I would consider as true, about cause and everything else, whatever agreed with this, and as untrue whatever did not so agree" (100a). This language is reminiscent of that which Plato will use in constructing the mathematical relations of proportional bonding in the *Timaeus* (31c), and suggests the possibility of metaphysical rather than merely logical connections. This suggests that perhaps Sayre was correct in emphasizing the ontic basis of the analytic connections.

There is of course good reason to compare this passage on the method of hypothesis with that of Pappus on analysis. Just like the double structure that he presented, this passage has two distinct and somewhat conflicting versions of how a hypothesis is developed. In the first half of the passage (101a), there is the reference to the accordance among the consequences of an hypothesis. This is reminiscent of the *Meno* passage as well as Pappus' reference to what "follows from" the initial presumption. It indicates a downward process.

The second part of the passage (101d), however, refers to a moving to higher hypotheses. This recalls the language in Pappus that analysis moves to that "from which" the hypothesis was drawn. The duality recognized in Pappus' statement of the analytic method can be seen to consciously preserve that same duality which is evident in Plato's presentation.

Robinson prefers to see all of these connections solely in terms of their logical significance. In his comprehensive study, *Plato's Earlier Dialectic*, Robinson attempts to reconcile these two passages of the *Phaedo* as a single, coherent presentation of Platonic logic.[34] Robinson closely examines the two alternative possibilities of interpreting the meaning of 'agreement' or as he translates, 'accords with.'

If we assume that 'accord' should be read as ' be consistent with' then we are forced to accept the truth of any proposition which does not contradict our hypothesis. Robinson believes this to be a "rash and unwarrantable thing to do."[35] He instead suggests that it is more reasonable to read accord as "is deducible from." This interpretation would be more in line with the language of 'follows' and 'consequences'

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[34]Richard Robinson, *Plato's Earlier Dialectic* (Oxford, 1953), pp. 115-45.
[35]Ibid., p. 126.

that we find in both the *Meno* and the *Phaedo*, but it involves an equally serious problem. Under this assumption, we would be forced to accept as false any propositions not deducible from the hypothesis. It would appear that we have to accept either this very narrow arrangement (implication), or one that is quite broad (consistency).

Robinson also concludes that there can be no third alternative. This condition forces him to choose between two less than perfect options. While admitting that reading 'accord' as consistency would be both less paradoxical and more consistent with other musical-like references in the dialogues, he opts to go the other way. The reason he gives is simple. If we read 'accord' as consistency, then he concludes that Plato would be assuming a logical impossibility: "for he would be assuming that the consequences of an hypothesis can contradict each other, whereas they cannot. The various propositions that follow from a given proposition are necessarily consistent both with the given proposition and with each other."[36] The way out of this corner is to realize that propositions which don't follow are not properly consequences. In this way he concludes that the operation of finding the consequences of an hypothesis just is implication.

Robinson does worry that such an interpretation puts an overly negative orientation on the method of hypothesis.[37] He consequently attempts to read the two *Phaedo* passages (100a, 101d) as different moments of one complex movement. The earlier passage only refers to the construction or development of an hypothesis. The latter refers to the checking of the consequences for consistency. In this way we can follow Plato's injunction: " not [to] jumble the two as the debaters do, by discussing the hypothesis and its consequences at the same time *(Phaedo*, 101d)."

But while Robinson recognizes two distinct moments in the method, he is equally convinced that both are inherently deductive: "You must first deduce the consequences of

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[36]Ibid., p.130.
[37]Ibid.

your hypothesis, and secondly deduce your hypothesis from a higher one."[38] Here again, as in the critique of geometrical analysis, Robinson seems to be unwilling to consider any other possible structures of connection than logical entailment (like causality or harmony).

1.3.2 Second Framework

The difficulty of accepting a double directional structure for the method of hypothesis has led some commentators to seek a second axis of critique, based within the framework of the upward or downward paths of the soul. Kenneth Dorter disputes Robinson's accounting of the method of hypothesis as deductive, and instead makes the case that it is essentially *dialectical*. Dorter, however attempts to distinguish within a broad definition of dialectic between division (*diaeresis),* and hypothesis, by emphasizing an element of *functional* directionality[39]. He focuses on the relationship between hypothesis and the Good to argue that the method of hypotheses works "upward" towards the Good: "The goal is the unhypothetical principle from which everything else follows."[40] Division, on the other hand, is the more technical analysis from first principles down to species. It is the "higher (more universal) for the sake of the lower (less universal)" and is "downward" in both orientation and emphasis: "The method of division, on the other hand, not only refrains from inquiring into the nature of the good, the highest principle, but the Eleatic stranger expressly warns that his method makes no distinction between greater and lesser value."[41] Dorter is arguing that the Good, precisely because it is beyond formulation within the Ideas, is that presumption which can only be hypothesized.

Dorter is considering the nature of a *functional* determination of directionality for the process of analysis. This possibility follows the argument behind Dorter's claim that

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[38]Ibid., p.141.
[39]Kenneth Dorter, *Form and Good in Plato's Eleatic Dialogues* (Berkeley), p.15.
[40]Ibid., p.14.
[41]Ibid.

the method of hypothesis, and therefore analysis, works towards the hypothesis of the Good. This examination ignores the particular kind of structural connections between the hypotheses and instead focuses solely on the end or purpose towards which they are framed. The functional directionality is not dependent on any particular, local relationship between individual propositions, but instead derives from the general *context* within which the hypotheses operate.

This reliance on directionality to distinguish clearly between the methods of division and hypothesis is problematic for two reasons. First, Dorter has reversed Plato's explicit association of dialectic as the "upward" path. In the *Republic* this reference towards dialectic as the way of philosophy, is directly contrary to Dorter's position. There is also the obvious clash with Robinson's reading of hypothesis as a ‘downward’ deduction.

This approach is also problematic in that the method of hypothesis seems fully capable of working in both directions - toward consequences and higher hypotheses. Dorter recognizes this duality, but chooses to emphasize one particular aspect of the process (the final movement) over any intervening moments. Equally, dialectic, which seems more explicitly the upward path, is at times not to be fully separable from the use of hypotheses. In the later dialogues dialectic is surprisingly applied to realms other than the Forms and we cannot always tell which way is ‘upward’.

Assuming that we can make some sense of this peculiar reversal of methods and objects, the next question becomes whether we can successfully locate just what the correct mathematical method of hypothesis is. Although Plato gives three reasonably detailed accounts of the hypothetical method - *Meno* (86c-87d), *Phaedo* (101a-d), and the *Republic* (533 a-d) - the metaphorical richness of his descriptions has led commentators to divergent conclusions rather than to converge toward a single definition. In his three presentations of the method Plato has emphasized three significantly distinct facets of this proposed operation – the making of an assumption, the mechanics of working back towards other hypotheses, and the working with images.

Clearly if we are to make sense of the similarities and distinctions between aspects of these two methods, dialectic and hypothesis, we must explore more closely those passages where these boundaries are Plato's main concern.

1.3.3 Third Framework

This further exploration, as represented in a third axis of methodological critique, is that of identifying the essential subject matter or object of the method, as taken from the Divided Line. This third exposition is the most explicit setting forth of the relationship between dialectic and hypothesis in any of the dialogues. The fact that it locates the distinction at least in part in the distinct kinds of objects to which each method naturally adheres weighs in favor of this criterion as a basis for finally distinguishing between the two methods. But I will claim that this move too would be premature.

Charles Kahn, in his work *Plato and the Socratic Dialogues*, develops this question of the differences between the two methods by working explicitly to decipher this maze of relationships behind and beyond the Divided Line.[42] Kahn has recognized that there is a distinction between dialectic and hypothesis which eclipses that of either logical structure or directionality and derives more directly from the question of the field of application of the method. As directed in the *Republic*, each method has its most appropriate realm, but can be utilized, secondarily, in other domains.

Kahn also goes beyond Robinson in recognizing the significance of distinct moments within the hypothetical method. In this way he can distinguish between that phase which is oriented toward the truth of the consequences drawn from a hypothesis and that which determines what in fact does follow. In this process Kahn's sensitivity toward Plato's usage of musical terminology (*sumphonein*), leads him to inquire beyond

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[42]Charles Kahn, *Plato's Socratic Dialogues* (Cambridge, 1996), pp. 312-320.

the strict logical models of deduction and entailment to wonder about what kind of connection Plato could have been pointing toward with his harmonic metaphors.

Parallel to the arguments of Cornford and Sayre on analysis, Kahn holds that the bond between related hypotheses is prior to and richer than those of mere entailment. He argues that Plato had a well developed sense of deductive entailment and consistency, but that the language of the method of hypothesis intentionally stays clear of such a commitment. His usage of ‘results from’ and ‘accords with’ is strictly restrained from committing Plato to an overly narrow view of these relations:

> Instead of speaking of what follows (*sumbainein*) Plato refers here to *ta hormethenta*, “what has proceeded” from the hypothesis, or “the sequel.” For a writer as careful as Plato, this choice of vocabulary must be significant. I suggest that the term for consequence is deliberately avoided, because Plato is here presenting the method of hypothesis as more flexible and also more fruitful than logical inference. The method functions not simply by drawing a linear chain of deductions but by building up a complex theory or constructing a model.[43]

Like Sayre, Kahn believes that there must be some richer, metaphysical basis for these connections. From these, all and any entailments could then be determined. The model for these ontic connections, as indicated by the Divided Line, is the realm of mathematics: “Just as mathematics makes use of visible models and diagrams in order to think clearly and consistently about intelligible structures, so dialectic makes use of mathematics itself as a conceptual model to achieve a knowledge of Forms.”[44] Although the mathematicians, like the generals, must turn over the fruits of their victories to those more capable of utilizing them, it is the mathematicians and their methods who “discover truths about reality (*Euthydemus*, 290c).”

These references to model construction in mathematics seem to support the speculations of Hintikka and Remes that hypothetical inquiry is a “synthetic” operation. This implication makes problematic the connection between the method of hypothesis

[43] Ibid., p. 316.
[44] Ibid., p. 295.

and analysis. And this is not merely a problem of the reversal of usage. If the method of hypothesis is a “constructive” activity, in what way can it be analytic? It seems that in order to make further progress on these issues we are constrained to lay out a proposal for the use of these apparently equivocal terms.

1.4 A Glossary of Usage

I propose to distinguish within Plato's usage between a broad or general sense, and a more narrow, technical sense in which he employs such terms as 'analysis', 'dialectic', and 'hypothesis'.

In this way "analysis," as taken from the most common agreement among Greek commentators, was generally taken to mean, “a process of resolution, of a whole into its parts, of a compound into its elements, of the complex into the simple.”[45] This idea could include the breakdown of either extended (part-whole) or conceptual (complex-simple) entities. Byrne finds this notion in Aristotle to mean a “loosening.”[46] This interpretation would seem adequate to capture either analysis of geometrical figures or of logical syllogisms.

Plato’s technical references to the "analysis" of the geometers, as we have seen, are more problematic. It will be one of the major goals of this dissertation to determine just what Plato considered the geometers’ analysis and then to understand how he in fact incorporated it into his own philosophical method.

The broad sense of "dialectic" coincides with those usages in Plato which encompass the wide spectrum of conversational dialogue from the negative applications of *eristic* and *enlenchus* of the earlier dialogues to that constructive inquiry more prominent in the later works. Sometimes it seems as if in Plato, "dialectic" is synonymous with "doing philosophy." What all these modes of inquiry seem to share in common is the sense of comparing or contrasting pairs of opposites. In eristic, such comparison is

[45]Gulley, p.4.
[46]Byrne, p. 11.

merely to refute. In elenchic, comparison is made to check hypotheses and purge inconsistencies.

For the technical usage of the term, I will limit the positive application of dialectic to the dichotomous division and collection referred to as *diaeresis*.

The broad meaning of "hypothesis," as Robinson informs us, comes from the Greek '*hupotithemi'*, "to posit in a preliminary way."[47] It is a derivative of '*tithemi'*, or "to stand." An hypothesis "conveys the notion of laying down a proposition as the beginning of a process of thinking, in order to work on the basis thereof."[48]

The technical meaning of what Plato means by "hypothesis" is more difficult to ascertain. In fact, as we have witnessed, the method of "hypothesis" seems problematically to contain three or four distinct kinds of procedures, only some of which can be uniquely attributable to the use of "hypotheses." As we mentioned earlier, we must minimally distinguish the distinct acts of 1) making the "hypothesis", 2) following the consequences of the hypothesis, and 3) developing succeeding, further "hypotheses." Some of these steps may involve activities which are "dialectical," such as the testing of the consequences for consistency with each other. To work out the nature of these hypothetical movements, it will be necessary to articulate fully the subtle distinctions which separate the moments of strict "hypothesis" from those of strict "dialectic" in that process which Plato refers to as dialectic in the more general sense. This will entail articulating the logical structure of the hypothetical procession.

Proceeding this way will facilitate the working out of some of the most obvious problems which we have identified. There are certainly moments of strict dialectic within that method which Plato calls hypothesis in the broad sense. In particular, both the drawing of consequences and the checking of them for consistency, are operations that are akin to a logic of dialectical rigor (elenchic).

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[47]Robinson, *Earlier Dialectic*, p.95.
[48]Ibid.

One use of "dialectic" involves the synthetic reconstruction of the results of the hypothetical analysis which puts it into deductive order. Syllogism can be considered a direct offspring of positive "dialectic" or *diaeresis*, since it is this categorizing of contraries which determines the ontological basis of the semantics of necessity. All categorical deduction assumes some sort of ontological hierarchy of class inclusion which undergirds the predication of essential properties.

A second dialectical activity is that linguistic inspection which determines whether two statements can in fact be held by the same person. This is the negative process of *elenchic*. Each of these operations must apparently work within the purview of the hypothetical method itself.

Third, there is the explicitly mentioned task of grounding those hypotheses which are finally most adequate. This is another dialectical operation involved with that of hypothesis, although it seems to function outside of the hypothetical analysis proper. This is the great promise held for dialectic in the *Republic* - the ultimate grounding of the mathematical assumptions.

Having identified those moments within the hypothetical method which are more properly entitled dialectic, we are in a better position to recognize those specific moves which are most properly hypothetical in a strict or technical sense. This is, after all, exactly how Plato first utilizes the hypothetical method in the *Republic*. It is only after eliminating the more recognizable virtues of wisdom, courage and moderation, that Socrates attempts to find that which is ‘left over’.

Since we have accounted for the drawing and the checking of consequences, as well as the possibility of grounding the highest hypotheses, it seems that the sole tasks that can possibly be identified as the essential feature of the technique of hypothesis, that is, strict hypothesis, are either the making of the hypothesis or the developing of further hypotheses.

I will defend the thesis that "analysis," is the general name for Plato’s comprehensive philosophical method. The structure of this method in its broadest interpretation, roughly follows the more technical orientation of geometrical analysis. The technical forms of both the method of hypothesis and dialectic will be identified as moments or phases of this larger process of analysis.

CHAPTER 2

THE ORIGINS OF ANALYSIS

2.1 The State of Ancient Mathematical Knowledge

One of the difficulties with attempting to reconstruct the ancient method of geometric analysis is what van der Waerden calls the 'holocaust' of extant works in the field.[1] Every single work from Pappus' *Treasury of Analysis*, except for the *Data* of Euclid, has disappeared. We have isolated, secondary references; we do not have a comprehensive and reliable account as to exactly what this method involved.

In his own efforts to penetrate this 'vale of tears,' Julian Coolidge attempts to examine exclusively those foundational questions such as "What do we mean by our fundamental terms?"[2] He utilizes an examination of some basic principles of mathematical thinking, as suggested by Gunther,[3] to support the position that it was the Greeks who invented geometrical analysis.

Coolidge's main contention is that "the essence of analytic geometry is the study of loci by means of their equations, and that this was known to the Greeks and was the basis of their study of the conic sections."[4] All the main technical achievements of the modern development of analytic geometry - the solution of quadratic and cubic equations, the use of higher degree curves, the finding of maxima and minima, as well as

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[1]B.L. van Der Waerden, *Science Awakening* (New York, 1961), p. 119.
[2] Julian Lowell Coolidge, *A History of Geometrical Methods* (New York, 1940), p. 45.
[3]van Der Waerden, p. 120 discusses Sigsmund Gunther’s 1877 work,
[4] Coolidge, p.119.

the utilization of the methods of exhaustion and equilibrium in approximating integration techniques - had some conceptual counterparts in ancient Greek geometry.

It is with the question of simplification of techniques where we find the most significant dividing line between the ancients and the moderns. It seems that the ancients came extremely close to the usage of a system akin to the modern algebraic symbolism, but seemed obstinately to refuse to take the "obvious" next steps.

A good example of this "ethos" is the Greek treatment of the algebraic system of the Babylonians, with which they clearly had a thorough acquaintance. Van der Waerden makes a convincing case that "geometric algebra" is a Greek development of the Babylonian system:[5] The Babylonians also used the geometric terms "rectangle" for xy and "square" for x^2, but besides these and alternating with them, such arithmetic expressions as multiplication, root extraction, etc. occur as well. The Greeks, on the other hand, consistently avoided the arithmetic expressions except in operations on integers and on simple fractions; everything is translated into geometric terminology.

Van der Waerden goes on to show exactly how the early Pythagoreans "formulated and proved geometrically the Babylonian rules for the solution of these systems [of equations]."[6] He concludes that the fact that all the normalized forms of these systems of equations, taken from the Babylonian cuneiform texts, appear in Euclid's *Elements* with their solutions, "gives clear evidence of the derivation of the geometric algebra of Book II from Babylonian algebra."[7]

It is also clear that the Greeks did not lack ability in the working out of algebraic situations. They were able to solve all quadratic equations through the combination of

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[5] van Der Waerden, p. 119.

the "application of areas" technique and the inscription of right triangles within the semicircle. The "application of areas" is a technique by which the area of a given figure can be converted to some other given figure with the same area. This technique is a geometric procedure that is equivalent to the more modern algebraic technique of solving first order equations.

The Greeks' ability to work with conic sections allowed them to resolve equations arising from the generation of "solid" loci, such as the "Delian" problem, the problem of doubling the cube. This famous problem was the initial recorded instance of the Greeks dealing with geometric problems that are equivalent to the algebra of cubic equations. Hippocrates' solution of finding two mean proportions between two given magnitudes is often used to represent the solution of any "solid" loci problems.

We can next address van der Waerden's tougher question, "Why did the Greeks not simply adopt the Babylonian algebra as it was, why did they put it in geometric form?"[8] Van der Waerden's simple answer is just that the Greeks had an unusual sense of the "enjoyment in what can be seen."[9] But I propose that this "quaint" view overlooks a much more important relationship between the development of Greek theoretical mathematics and its peculiar, yet essential alliance with philosophy.

The significant interaction between philosophy and mathematics at the Academy can be appreciated through a quick perusal of the number and stature of mathematicians who were active in the life of Plato's *padeia*. The development of all of the work in

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[6] Ibid., p. 124.
[7] Ibid.
[8] Ibid., p. 125.
[9] Ibid.

Euclid, as well as much which appears in Apollonius and Archimedes, can be traced back to mathematicians with direct contact with the Academy.[10]

What this extensive interaction implies is that in addition to the issue we are examining, that of the direct influence of geometric techniques on the philosophical method of Plato, there likely was concurrently an inverse influence of Platonic philosophic method on the self-conscious development of the methods of inquiry of the mathematicians. With this possible double movement of interaction in mind, we can perhaps begin to see a more direct relationship in the applications and development of the two great systems of the Academy: the “royal art” (dialectic) and the “royal road” (analysis).

Analysis supposedly received its denomination as the "royal road" in the time of Euclid, in the generation directly following Plato's at the Academy. Apparently, Ptolemy II asked Euclid if there were any "shorter way" than that of the *Elements* to master geometry, and that Euclid answered that there was no "royal road". Whether or not analysis was known as a royal road at the time of Plato's Academy, some of its techniques represented the possibility of a simpler or shorter path to the solving of certain kinds of geometric problems. In particular, the reducing of cubic problems in geometry to relationships between conic section curves and then solving those relations with mechanical or numerical devices, amounted to a kind of modern algebraic analysis.

The idea that analysis was a Royal Road implied that there were "short cuts" that could make complex and difficult geometrical problems accessible by mechanical or arithmetic techniques to practitioners who did not necessarily have to be fully in

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[10] Ibid. , pp. 148-200.

command of the conceptual intricacies behind the geometry. This reference to an abbreviated method of attaining knowledge is reminiscent of the warning that the Stranger in the *Statesman* offers about commanding the art of dialectic: One must first master the way of "longer division", before pursuing the shorter path of cuts. The influence of philosophy on the mathematicians was not so much to eliminate numerical techniques from geometry as to emphasize that the conceptual relations behind such simplifications must remain a priority.

The prioritizing of this relationship between the longer and shorter paths can help us to sort out one other seemingly paradoxical set of anecdotes passed on about Plato. The Delian Problem of doubling the cube is one that is considered a dominant theme in the development of Greek geometry, and it shows up in many interesting places in the dialogues. This problem was solved by each of the great mathematicians of the Academy in a unique way. Archytus solved the problem, which Hippocrates had reduced to that of finding two mean proportions, by the application of semi-cylinders. His student Eudoxus utilized the intersection between circular cylinders to demonstrate how the indeterminate relationships could interact in such a way as to produce a determinate solution. Eudoxus' student Menaechmus utilized the interaction between the conic sections of the parabola and hyperbola to accomplish the same[11].

An inconsistency arises from the diverse anecdotes relayed as to Plato's own attempted solution to this problem. Plutarch relates that Plato was highly critical of all the previously mentioned efforts, since they all involved the utilization of mechanical

[11] Heath, p.287.

tools beyond the ruler and compass to construct the higher curves needed[12]. This criticism is easily accepted by commentators since it seems to stress the theoretical outlook which it is assumed that Plato always adopted. However, Eutocius, in his commentary on Archimedes, tells us that Plato resolved this problem with the invention of a specific mechanical tool, involving a builders square with a movable second ruler, to solve the problem[13]. Which story are we to believe?

The issue is less problematic in light of our distinction between a longer theoretical approach and a shorter, practical road. The use of a mechanical device is warranted and useful once one has mastered the conceptual relationships which ground and are the theoretical basis for the mechanical tool. In this way the tool itself is a kind of theoretical implement.

But how does this resolution of the theory-practice debate affect our interpretation of whether the tools of geometrical analysis could have become a kind of "royal road"? As aspired to by its inventors, Fermat and Descartes, the methods of modern analytic geometry certainly approach a kind algorithmic status, which no longer depends on the ingenuity or intuition of the user to command the solution of difficult problems. Equations and curves are classified in terms of their characteristics and each has its own mechanical technique of solution.

The resistance of the Greeks against adapting such a fully formal approach to analytic geometry can possibly be accounted for by the influence of philosophy in its emphasis on the activity of *mathesis.* The "simplifying" of mathematical manipulation also contributes to blinding the user to the true roots of the techniques. Much like his

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[12] Ibid., from Plutarch, *Quaest. Conviv*. viii. 2. 1, p. 718 F.

complaints against writing in the *Phaedrus* and the *Seventh Letter*, Plato surely believed that the relieving of the soul of the difficulty to visualize complex geometrical relations could undermine mathematics' ultimate benefits for the soul - that of conditioning the virtue of genius.

2.2 Locus Problems and the Determination of Paradigm

Key to understanding the nature of geometrical analysis is appreciating the unique qualities of locus problems. The second problem of the *Meno* (86d-87c) is a "locus theorem". Such a problem is one determined by a "continuum" or "locus" of points determined by a given condition. Most problems involving circles and parallel lines are locus problems, for these figure constructions are given by a single, guiding condition (distance from a center; alternate interior angle equality).

The simplest example by which to understand what is involved in a locus problem is that of a locus construction. The Fig. 2.1 is an example of circumscribing a circle around a given triangle illustrates the basic properties of a locus construction.

In this problem, in order to construct the circle which coincides with the three vertices of the triangle, we must determine that the center of that circle, 'O', will be equidistant from the three vertices of the triangle. This condition, that of finding a point equidistant from three other points, may in turn be translated into another set of locus constructions. To find the locus or set of points equidistant from two given points, one needs only to construct the perpendicular bisector of the line between the given points.

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[13] Ibid.

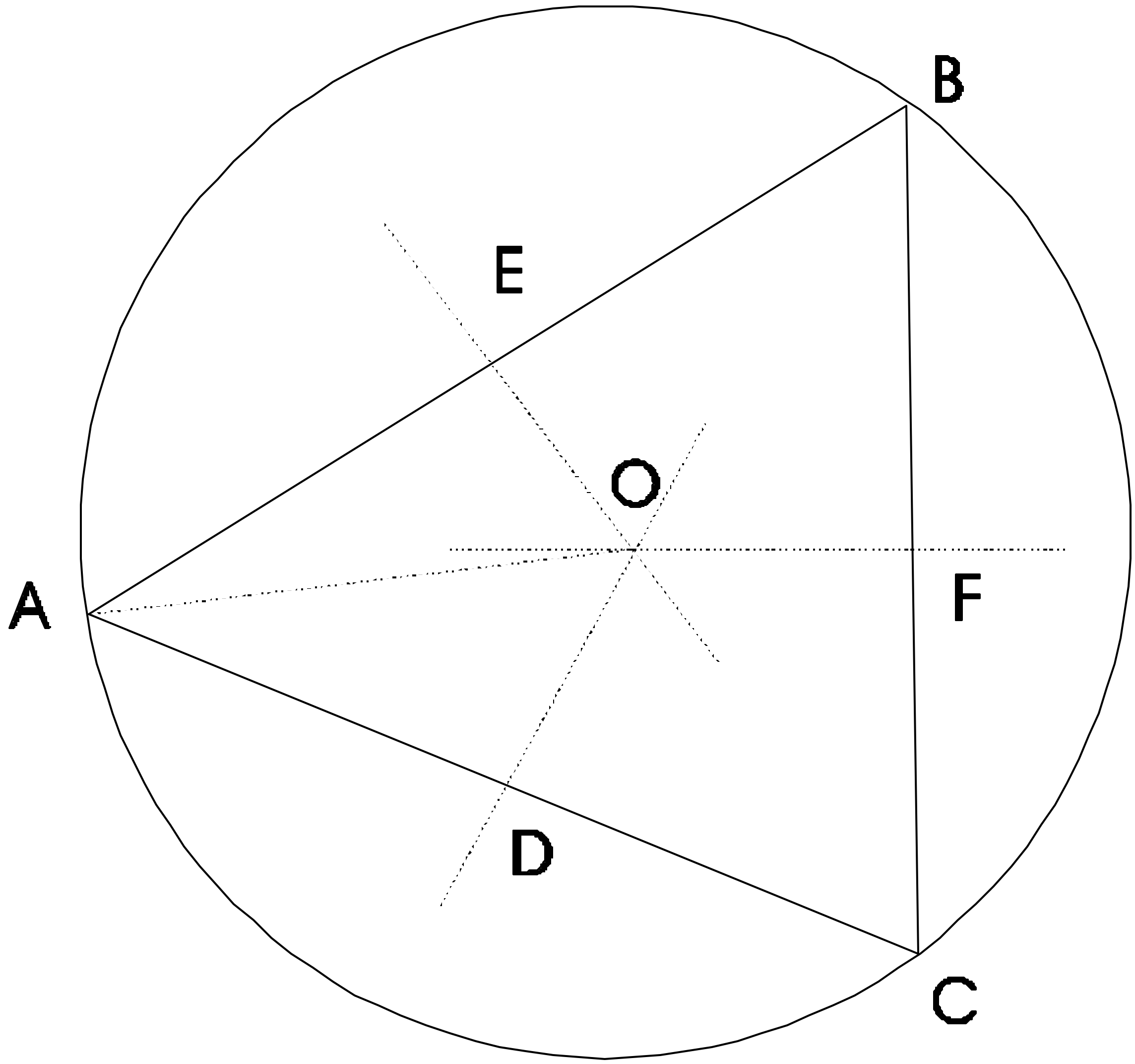

Fig. 2.1

So OE, as the perpendicular bisector of AB, represents all the points which are the same distance from both A and B. Now the center of the circumscribed circle will be that point which is the intersection of any two of these perpendicular bisectors, since such a point would by the conditions of construction then be the same distance from all three points.

Mathematically, the example is straight forward, but it can be understood as the application of a more general technique. One locus of points, the circle, was found by determining the intersection of three other loci of points, which determined the circle's center.

The determined connections between these two sets of loci best illustrates Chrysippus' comments on the relationship between locus problems and Plato's Theory of Ideas. Proclus informs us that Chrysippus "likened theorems of this sort to the (Platonic) Ideas."[14] Like Ideas, the necessary connections of the loci are fully transparent to reason. Not only can we 'see' the condition of equidistance in the construction of the perpendicular bisectors, but the relationship between this construction and that of the circumscribed circle is easily translatable into the principles which determine the definition of the circle. The circle, determined by its center and the distances to the separate vertices, reflects the equality of the perpendicular bisectors of the sides. This "transitivity" between levels of a locus problem illuminates how concepts may be represented through such universal convertibility. Since each locus construction is interpretable in terms of a single principle or condition (ie. the equidistance from two points of the perpendicular bisector and its relationship to the definition of a circle), the relationships between the constructions just are those between the discursively related principles.

This simple locus construction the anc ients termed a 'plane' locus, since it could be fully determined by use of the tools of plane geometry, the compass and the straight edge. They also examined two other levels of locus problems, the 'solid' and 'linear' loci. Solid loci are those which require the use of certain higher curves to solve. Since these

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[14] Proclus, *A commentary on the First Book of Euclid's Element,* (Princeton 1992), p. 311.

curves could all be determined by the cross sections of the solid cone, they are referred to as solid loci.

The third and most complex sort of loci are the linear loci. Descartes developed his coordinate geometry specifically to solve a linear locus problem raised by Pappus[15]. The famous Four Line Locus of Pappus was a reconstruction of a problem solved by Apollonius[16].

In the *Meno* problem, Socrates makes reference to the kind of problem by which geometers often investigate their hypotheses:

> For example, if they are asked whether this triangle can be inscribed within this circle one of them might say: I do not yet know whether this triangle is of that kind, but I think I have, as it were a hypothesis that is relevant to the problem, namely this: If this triangle is such that, when laid along the given line it is short by a space as large as the figure laid along it, then I think one thing follows, whereas another thing follows if this cannot be done. So I am willing to tell you on this hypothesis about inscribing it in the circle, whether it is impossible or not (Meno, 87b).

Such a problem begins with two unknowns: what the figure is and what its qualities may be. Locus constructions represent indeterminate relationships between the given variables. As such they are a graphic representation of our "ignorances": what the figure is and what its qualities may be. Yet somehow this method of loci can produce a kind of knowledge from a type of ignorance.

Locus problems are often not amenable to solution through any other standard analysis by geometrical techniques such as congruency proofs. They do not assume a construction, but attempt to determine the conditions for the possibility of one (diorism).

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[15] David Lachterman, *The Ethics of Geometry* (New York, 1989), p.144.
[16] Ibid., p. 146.

For philosophers with mathematical interests, these problems stirred a great interest for their relationship to the grounding of mathematical thought.

Proclus, in his Commentary on the First Book of Euclid's Elements, relays to us three distinctive marks of locus theorems. First, Different than the triangular construction problems of the first half of Book I, locus constructions are determined by " a position of a line or surface producing one and the same property."[17] As such these constructions were paradigmatic of the activity of participation, "for just as the Ideas embrace the generation of an indefinite number of particulars within determinate limits, so also in these theorems an indefinite number of cases are comprehended within determinate loci."[18] Proclus shows us how this locus idea can model such paradoxical phenomenon as an infinite number of different parallelograms which can each be applied to the same base between the same parallel lines, all with the same area. The perimeter of these figures can increase indefinitely without any increase of area: "Their equality is shown to result from this limitation; for the height of the parallels, which remains the same while an indefinite number of parallelograms can be thought of on the same base, shows all these parallelograms to be equal to one another."[19]

Proclus' second comment is even more illuminating. In going over the many examples of locus theorems involved with parallel lines, Proclus points out that the three varying conditions of the proofs - the equality of the bases, the sameness of the parallels and the equality of the figured areas - are all interchangeable in creating various possible conversions[20]. What he seems to be asserting is that locus theorems all have the nature of

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[17] Proclus, p. 310.
[18] Ibid., p. 311.
[19] Ibid., p.312.
[20] Ibid., p. 313.

quantity for quantity conversions. Different types of figures that are all of the same magnitude can be conditionally equated with each other. More specifically, any rectilinear figure, smaller than a given equilateral triangle, can be inscribed in a given circle (*Meno* problem).

This particular locus problem demonstrates the convergence of two distinct heuristic structures. The first, a *neusis*, is the reduction of a problem with an unknown solution to one with a solution already worked out. This method had been prevalent in geometry since Thales. The presence of such a problem at this crucial juncture in a dialogue stressing the importance of method, shows the interest within the Academy about the nature of how such problems worked.

The reduction represents a neutral operation in the nature of geometrical thinking. It is neither ampliative in directly increasing knowledge about the figure, nor is it universally objective in giving the determinate relationships between the transposed figure and the constraining conditions of the construction (i.e. line, angle, etc.).

The second aspect of this locus problem is a diorism. A diorism is a way of setting up a potential problem to determine beforehand whether or not it can be resolved. It determines the conditions for the possibility of a problem's solution. Proclus recounts that one of Plato's students, Leon, first developed diorism as a formal method.

It is the heuristic dimension which leads us to Proclus' third comment on the nature of locus problems. He points out that the converse proofs of particular locus theorems are always indirectly demonstrated. For example, there is apparently no direct means of demonstrating that two independent sets of conditions (such as the constraints determining two distinct lines) can in fact determine that their constructions are one and the same (a single line). So when Euclid, in Proposition 39 wants to prove that "equal triangles which are on the same base and on the same side are also in the same parallels", the converse of Proposition 37, he is forced to show that the line joining the vertices of the two triangles "can't be different than the given parallel", but not directly that it is the same[21]. This logical "blind spot" in these kinds of proofs has been little discussed by the tradition. This inability by logical means to directly confirm some of the critical conditions for construction, is what both Aristotle and Hintikka[11] refer to as the strange appropriateness of analysis for "figurative" kinds of knowing. There is some essential component of the figurative schema (a semantic possible element) which seems to evade the constraints of any directly applied logical syntax.

The second mathematical problem of the *Meno* is not nearly so difficult as the Pappus Locus Problem. It is instead a 'solid' locus problem. The problem is to determine whether a certain quadrilateral of area 'b^2' can be given as the area of a triangle inscribed in a given circle of radius 'a' (86e).

As an analysis, we first assume that this can be done (specifically as an isosceles triangle[22]). We then must show that the constructed isosceles triangle is equal in area to a rectangle constructed on the diameter such that the rectangle on the remaining portion of the diameter is similar to the one first constructed. This characteristic is the equivalent of showing that the "half base" of the isosceles triangle is the mean proportional between the segments of the diameter (because it is the altitude of a triangle inscribed in a semicircle)(Fig. 2.2):

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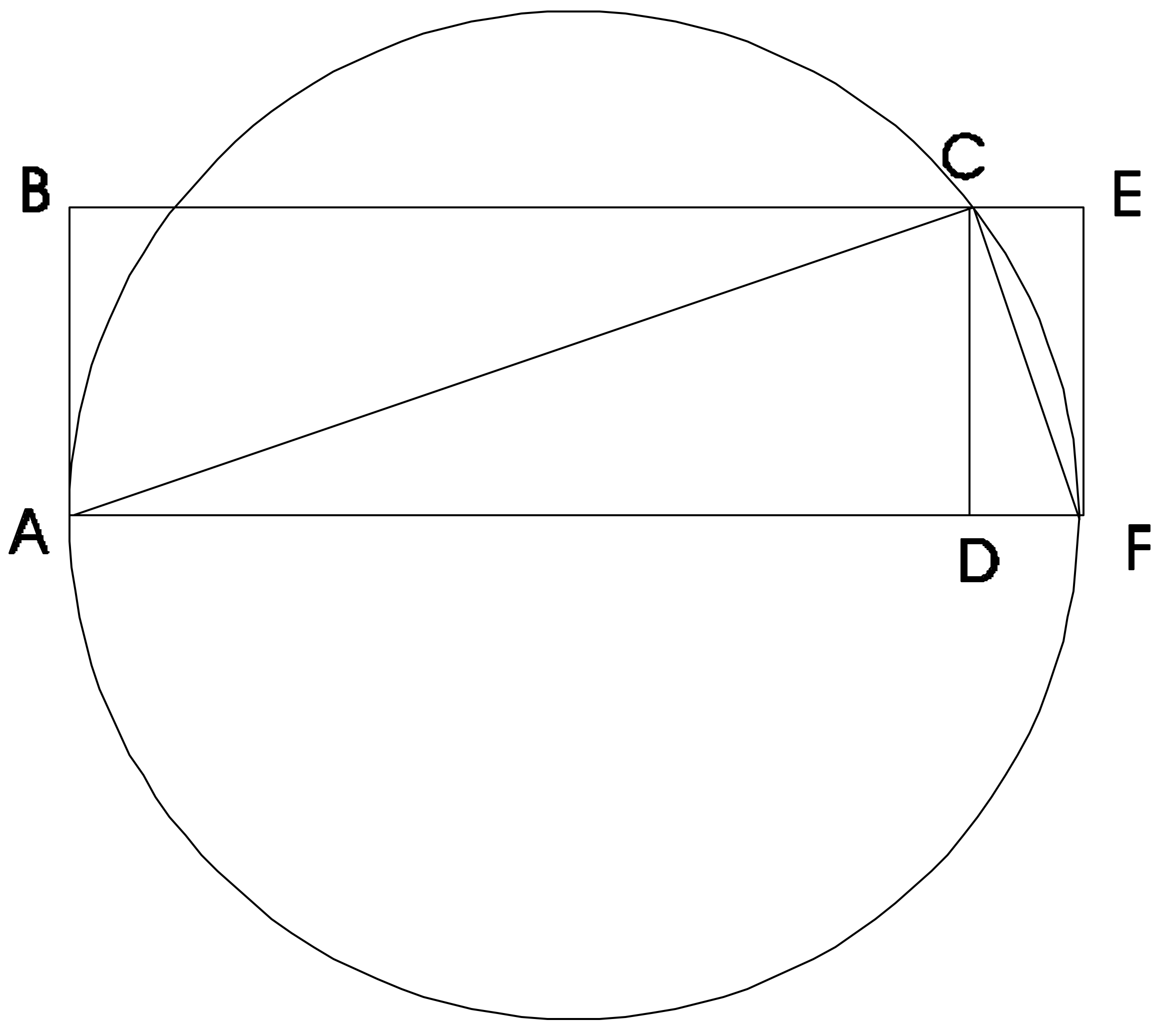
[21] Euclid, *The Thirteen Books of the Elements, Vol. I*, ed., T. L. Heath (New York, 1956), pp. 332-8.
[22] Although any given triangular area can be constructed as an isosceles triangular area, the question of whether the scalene area is always less than the isosceles area is not answered until after our analysis is completed - with the equilateral being shown to be the greatest triangular area.

Fig. 2.2



The isosceles triangle (ACG) will always be bisected by the diameter (AF) of the circle. In other words, the two halves of the isosceles triangle can be said to be equal to a rectangle (ABCD) constructed onto that diameter.

By this construction we can see that the given condition can be satisfied by all rectangles one of whose sides lies on the given circle (AD), with one pair of opposed vertices (A, E) situated on the circumference.. This condition means that angle ACF will always be a right angle and the altitude, CD, of the inscribed right triangle will always give us the required proportionality between the lines, AD:CD::CD:DF. So our original problem, that of applying an area as a triangle inscribed in a given circle, has been reduced to that of applying the original rectangular area to the diameter of that given circle, in such a way that the applied rectangle (ABCD) and the rectangle constructable on the remainder of the diameter (CEFD), are similar. This similarity is given by the relationship of mean proportionality.

This is where the difficult work begins. In order to determine the areal conditions under which such mean proportionality will be possible, it is necessary to find that limiting value for which they will not attain - the maximum beyond which failure results. That is, it must be determined which isosceles triangle has the largest area. Intuitively, it is obvious that the triangle which meets this requirement is the equilateral: however, the problem is to prove the same.

The way in which this feat is accomplished is to realize that if one considers one vertex of the isosceles triangle as lying on a rectangular hyperbola ($xy = b^2$, where b^2 is the required area), and another as fixed at the origin, then the intersections of the different possible hyperbolae with the given circle ($x^2 + y^2 = 2ax$) represent those points where a second vertex of an isosceles triangle can be located to generate an inscribed isosceles triangle with area b^2, such that the first vertex, located at the origin, is such that the line connecting the two vertices (AE) indicates one of the two triangular sides of equal length.. The triangular area is maximized at the point where there is only a single solution, where the hyperbola is exactly tangent to the circle. This solution is the equilateral triangle (Figure 2.3).

In the figure, triangle AEG is constructed to be equal in area to the given rectangle, $x=b^2$. The hyperbolic curve, EE' gives all the rectangular solutions (two or one) for a given area.

This problem is equivalent to solving an equation of the fourth degree ($x^2 [2ax - x^2]= b^4$)[23]. This second problem is the equivalent to finding two mean proportions between two given values. Although some mathematicians dispute whether Plato had the means of solving this problem at the time of the writing of the *Meno*[24], there are good reasons to believe that Plato was referring to this very problem.

The major factor behind this assertion is the closeness of the wording of the *Meno* problem and one similar to this in Euclid (VI, 29): "To a given straight line to apply a parallelogram to a given rectilinear figure and exceeding by a paralleogramic figure similar to a given one[25]." The problem in Euclid is exactly parallel to the one we have reconstructed, with the exception of using a rectangle instead of a parallelogram.

Then there is also the common theme of finding two means, which surfaces prominently latter in the *Timaeus*. There Plato asserts that between two "solids", it is

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[23]. [54]Heath, p. 301.
[24] Ibid.
[25] Euclid, p.265.

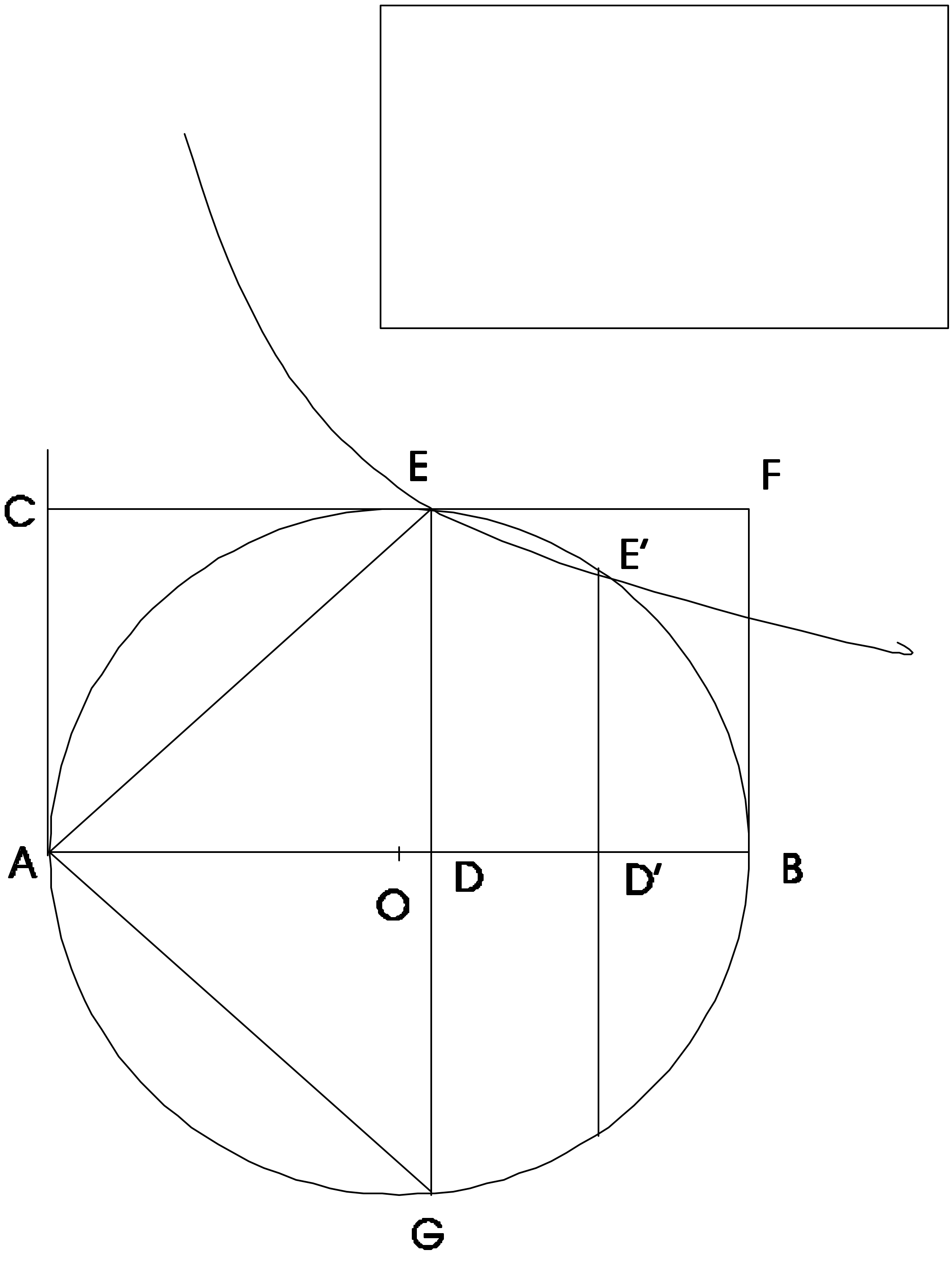

Fig. 2.3

necessary that there be two means as bonds (Plato, *Timaeus*, 32b). It is this condition which establishes the necessity for there being four elements. Now most people analyze this problem in the *Timaeus* as one of merely developing a continuous proportion with four terms[26]. But the problem is specifically set up as one of finding two means between two given magnitudes.

Just like finding a single mean proportion was the equivalent of solving a quadratic equation, and applying figures was the equivalent of setting areas in equation, the finding of two mean proportions was the way in which Greek mathematicians typically solved cubic equations[27].

There is also the fact that this problem involves the same solution as another which Plato is reputed to have solved - the Double Cube or Delian Problem[28]. The poetic possibilities which are evoked in this suggested parallel between the Double Square (first problem) and the Double Cube (second problem) pull strongly toward Plato's sense of play. This relationship between the dimensionality of the two problems is further reconfirmed by the history of mathematical problem solving at the Academy. One of the most written about problems was that of the Doubling of the Cube. This is a problem whose distinct solutions have been attributed to Archytus, Menaechmus and Plato himself[29]. All of the solutions involve different ways in which to find two means between two given magnitudes. Archytus' solution specifically involves the utilization of curves in three dimensions, and is termed a "solid" locus problem. Analogously, the two

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[26] Francis Cornford, *Plato's Cosmology* (New York, 1957), p. 47.
[27] Heath, *History*, p. 252.
[28] Ibid., p. 287.
[29] Ibid., p. 245.

problems in the *Meno* are examples of using plane and solid loci problems for finding the double of plane and solid figures.

4.31 On Porisms

With almost the same unanimity with which modern philosophical studies of Greek analysis have ignored the phenomenon of "porisms", the great mathematicians of early modernism had esteemed them. One may go so far as to state that the major impetus from which modern analysis emerged was just that which arose from the reconstructing of this and other ancient methods. Yet in the recent works which have examined ancient Greek analytic method, only a single author has so much as mentioned this lost art of analytical thinking (Mahoney)[30].

Both Fermat and Newton, as well as many others, attempted detailed reconstructions of the lost Euclidean work on porisms[31]. They each claimed to have "discovered" something in the work of the ancients which was purposefully obscured by the authors and of tremendous value for their own developments of mathematical analysis.

While it is clear that the ancients relied on the poristic method in a much more comprehensive and systematic way than the early modern re-inventors of analysis, we can only speculate, from fragments and ancient commentators, exactly what their method was. There is no direct evidence whatsoever, as to the practice and nature of poristic analysis. It is almost as if there were a purge or systematic elimination of this particular kind of mathematical thinking which took place some time in the Hellenistic age.

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[30] Mahoney, *Fermat*, p. 30.
[31] Euclid's work, *On Porisms* is one of the lost works of analysis from Pappus' *Treasury*. See Bibliography for list of reconstructions.

The indirect evidence that we do have is extensive, but somewhat confused. Pappus and Proclus are our richest sources. Their accounts, while somewhat repetitive, still leave many of the important details of the method, unknown.

In an extract from the seventh book of his Mathematical Collections, Pappus gives us the following concerning the three books of Euclid's *Porisms*[32]:

> After Tangencies we have in three books, Euclid's Porisms, a most subtle collection of many things which relate to the analysis of the more difficult and general problems, of which indeed nature affords a great plenty....Now there contains a subtle and natural theory, very necessary and universal, and highly entertaining to those who are able to understand and investigate each. Now all these are in species neither theorems or problems, but in form sort of a middle nature between them; so that their propositions may be formed either as theorems or problems. From whence it has come to pass that among many Geometers some have esteemed them in kind to be theorems, and others problems, having respect only to the form of the proposition. But it is manifest from their definitions that the ancients better understood the difference between these three. For they said that a theorem was that in which something was proposed to be demonstrated; a problem that in which something was proposed to be constructed but a Porism was that in which something was proposed to be investigated. Now this definition of a Porism is changed by the moderns, who could not investigate all these, but using these elements, showed this only what it was that was sought, but did not investigate it. And though they were confused by the definition, and by what has been said, yet they formed their definition, from an accident, in this manner. A Porism is what is deficient in hypothesis from a local theorem. [i.e. A Porism is a locus theorem deficient or diminished in its hypothesis.] Now of this kind of Porism geometrical loci are a species, of which there is great store in the books concerning analysis, and being collected apart from the Porisms, are delivered under their proper titles, because this species is much more diffuse and copious than the rest. For of loci some are plane, some solid, some linear, and besides these there are loci ad medietates [or which arise from mean proportions.] This also happens to Porisms that they have concise propositions because of the difficulty of many things which are wont to be understood; from whence it comes to pass that not a few Geometers understand the matter but in part, whilst they do not take in those things which among the things shown are more necessary. Now many of them can by no means be comprehended in one proposition, because Euclid himself has not put down many in every species, but that he might give a specimen of great copiousness, at the beginning of his first book he has put down a few altogether of the same sort with that most fruitful species of loci [which is

[32] I will utilize some quotes that are larger than usual in order to convey the nature of these very difficult conceptual relationships.

> the subject of his first book] so that they are ten in number. Wherefore perceiving that these [viz. Propositions of this species] may be comprehended in one proposition, we thus describe it[33].

Pappus goes on to talk about some of the specific propositions which Euclid included among the porisms. In this summary he comments about that which distinguishes them from other kinds of analysis:

> Now it is not likely that Euclid was ignorant of this, but that he had respect to principles only: for through all the Porisms he seems only to have sown his first principles and seeds of many and great matters. Now these are by no means to be distinguished according to the differences of the hypotheses; but according to the differences of accidents and things sought. All hypotheses indeed differ from each other, as they are most special; but every one of the accidents and things sought, since it is one and the same happens to many and various hypotheses[34].

Pappus' remarks on porisms bear striking similarity to Proclus' remarks on the same subject, and the latter may well have been his source. Yet there are interesting differences.

Proclus first reviews the term in his Commentary on the First Book of Euclid's Elements:

> "Porism" is a term applied to a certain kind of proble m, such as those in the Porisms of Euclid. But it is used in its special sense when as a result of what is demonstrated some other theorem comes to light without our propounding it. Such a theorem is therefore called a "porism" as being a kind of incidental gain, resulting from the scientific demonstration.[35]

He soon after elaborates on this brief introduction:

> "Porism" is a geometrical term and has two meanings. We call "porism" a theorem whose establishment is an incidental result of the proof of another theorem, a lucky find as it were, or a bonus for the inquirer. Also called "porisms" are problems whose solution requires discovery, not merely construction or simple theory. We must see that the angles at the base of an isosceles triangle are equal, and our knowledge in such cases is about already existing things. Bisecting an angle, constructing a triangle, taking away or adding a length - all these require us to make something. But to find the center of a given circle, or the greatest common measure of two given commensurable magnitudes,

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[33] Heath, *History*, pp. 43-44, from Hultsch translation of Pappus, vii. Pp. 648-60.
[34] Ibid.
[35] Proclus, p. 166.

and the like - these lie in a sense between problems and theorems. For in these inquiries there is no construction of the things sought, but a finding of them. Nor is the procedure purely theoretical; for it is necessary to bring what is sought into view and exhibit it before the eyes. Such are the porisms that Euclid composed and arranged in three books.

A porism, then, is a theorem whose truth becomes evident without effort through the proof of another problem or theorem. For we appear to hit upon porisms as it were by accident, not as answers to problems or inquiries; hence we likened them to lucky finds. And it may be that the masters in mathematics gave them this designation in order to show ordinary people, who get excited over some apparent gain, that these, and not the sort of things they suppose, are the true windfalls and gifts of the gods. For they are produced by the resources we have within us; our prolific capacity for knowledge adds them to the results of the preceding inquiries, thus revealing the inexhaustible richness of the world of theorems.

Such, then, is the way in which the peculiar character of porisms is to be described. They can be classified, first, by the sciences in which they appear: some porisms belong to geometry, others to arithmetic. The one before us is geometrical, that which occurs at the end of the second theorem in the seventh book arithmetical. Secondly, by the propositions which precede them: some follow problems, others on theorems. The present one results from a theorem, but that in the second book, comes from a problem. Thirdly, according to their methods of proof: some are established by direct proof, others by reduction to impossibility. The one before us is made evident by direct proof, whereas that which is implied in the proof of the first theorem of the third book comes to light by a relation to impossibility. There are many other ways of classifying porisms, but these are enough for us at present.

The porism that we are now discussing, in teaching us the space about a point can be divided into angles equal to four right angles, forms the basis of that paradoxical theorem which proves that only the following three polygons can fill up the space about a point: the equilateral triangle, the square, and the equilateral, equiangular hexagon.

This porism also enables us to prove that, if more than two lines - three, or four, or a number you like - cut one another at a single point, the angles that result will be equal in sum to four right angles, for they divide up the space of the four right angles. It is clear also that the angles will always be double in number or the straight lines.[36]

[36] Ibid., pp. 236-7.

Proclus makes mention of two very distinct usages of the term porisms. The one, translated as "corollary", is just that accessory consequence which often accompanies the working out of a proof. The fact that this extra knowledge is "accidental" to the original proof seems to support a belief about porisms that they are representative of the blind or intuitive nature of mathematical discovery.

The second understanding of the term is quite different. A porism is a directed or intentional discovery. Poristic reasoning is a systematic approach to finding unknown truths. If these two usages are to be compatible, then we must understand the meaning of corollary as an accessory consequence which gives some sort of insight into the nature of a problem or theorem.

This ancient fascination with porisms would perhaps be of less direct interest, were it not for the profound effect that these propositions have had on the great minds of the modern era. As previously noted (p. 58) Fermat and Newton, both spent a considerable amount of time attempting to reconstruct the poristic system of Euclid. While Fermat's work remains untranslated, it is clear from his references in other works that his reconstruction of poristic was a mainstay in his development of modern analysis. Newton's comments illustrate the extent to which he took this kind of analysis to be the key to unlocking the ancient secrets of mathematical technique.

In this project, Newton makes some interesting comments about the nature of poristic analysis. He notices that ancient techniques of analysis form an art whose secret "lies hidden to geometers of our time."[37] He also notes that a porism is a proposition "whereby out of the circumstances of a problems we gather some given thing of use to its resolution."[38] In one of his reconstructive works, "The Finding of Porisms", Newton writes:

> It takes the form either of a theorem or a problem at pleasure, and is in consequence reckoned to be a sort of mean between each. The given things, however, which are to be thus gathered are direct and inverse proportions and other relationships of unknown quantities; likewise the species of figures and the lines in which unknown points are located, and which in consequence are usually said to be the loci of the points; and also lengths, angles and points which either regard the determination of loci or otherwise contribute to the resolution of a problem. But proportions hold first place and are tracked down by means of the following theorems:[39]

Newton eventually reconstructs what he takes to be the five theorems which demonstrate the nature of poristic reasoning. These principles are interesting for two reasons. First they seem to go well beyond anything directly implied by the commentaries of either Pappus or Proclus. Second, one can see in these principles the seeds of Newton's own methods in the development of the calculus. In particular, there is the emphasis of the kind of dynamic interaction which seems to emerge between different kinds of indeterminate quantities, as given in the first of the five principles:

> 1. If two indeterminate quantities, or their uniform powers, mutually determine each other simply, and these either simultaneously vanish and simultaneously become infinite, or simultaneously vanish and once return to the same ratio, or finally if they twice return to the same ratio, then they will be in a given ratio
> 2. But should it happen that each successively vanishes when the other becomes infinite, or if one of them vanishes when the other becomes infinite and both return once to the same mean proportional, or finally if they twice return to the same mean proportional, then they will be reciprocally proportional.
> 3.If two indeterminate quantities, or their uniform intermediate powers mutually determine each other doubly, and certain ‘planes’ (quadratic products) separately made up out of them twice simultaneously vanish and simultaneously prove to be infinite, or should one or more of these four circumstances be lacking and the

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[37] Isaac Newton, "The Finding of Porisms", *The Mathematical Works of Sir Isaac Newton* (New York, 1964), p.231.
[38] Ibid.
[39] Ibid.

> deficiency be filled by an equal number of returns to the same ratio, then these ‘planes’ will be in a given ratio.
> 4.But if each successively vanishes when the other comes to be infinite, and that twice, or if one or more of these four circumstances be lacking and the deficiency be filled by an equal number of returns to the same mean proportional, then the mean proportional of the ‘planes’ will be given. And so you may go on indefinitely.[40].

Newton's approach here is important to take note of. He is taking what I consider to be a paradigmatically poristic approach to analysis. Envisioning himself as a Baconian explorer, torturing nature for her secrets, Newton is systematically utilizing the two kinds of "failure" that result from an indeterminate relationship, impossible solutions and unlimited solutions, to objectively determine the characteristics of that same relationship. This procedure will eventually be reduced to the algorithms of algebraic analysis. But at this point, by isolating and identifying each separate kind of possible relationship, as it specifically relates to a set of geometric, indeterminate figures, Newton stands much closer to Archimedes and Menaechmus than to modern analysis. We should notice that this is not a blind process of falling into aporia just in order to wander across some brilliant means of solution. Poristic analysis is a "controlled fall" into enigma, where the precise steps leading to the failure systematically become the positive map of transformation.

The influence of ancient analysis on the mathematical works of Fermat and Newton did not go unnoticed among the commentators who came after these mathematicians. Robert Simson (*A Treatise Concerning Porisms*)[41], and William Playfair (*On The Origin and Investigation of Porisms*)[42] wrote extensively on the nature

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[40] Ibid., p. 234.
[41] Robert Simson, *A Treatise Concerning Porisms* (Canterbury, 1777).
[42] John Playfair, "On the Origin and Investigation of Porisms", *Works of John Playfair, Vol. 3* (Edinburgh,

of porisms. Each attempted to compare both ancient and modern sources, as well as to reconstruct the nature of the ancient porism. Playfair has the most probable and subtle account and it is the one I will discuss here.

Interestingly enough, Playfair begins by justifying the very project of a reconstructing of ancient mathematical works. He gives two reasons why the ancient structure of porisms may finally be rediscovered. First he cites the Collection of Pappus and its comments and insights. But then, and more importantly, he states the principle by which such an activity might succeed: "The second is, the necessary connection that takes place among the objects of every mathematical work, which, by excluding whatever is arbitrary, makes it possible to determine the whole course of an investigation when only a few points in it are known."[43] In other words, Playfair is going to utilize the very nature of poristic investigation to reconstruct what it is that porisms precisely were.

Playfair also offers useful criticism of previous attempts to make this same reconstruction. On Pappus' two definitions of porisms, he criticizes the first (a local proposition diminished by a hypothesis), on the grounds that it is itself rightly rejected by Pappus as imperfect, while the second (a thing sought, but needing some finding, being neither brought into existence simply nor yet investigated by theory alone), is "too vague and indefinite to convey any useful information."[44] He also takes Fermat's work to task for accepting the first definition of Pappus, despite Pappus' own criticism of it as due to "junior geometers". He holds that Fermat neither proved that any of his own concept of porisms were that of Euclid, nor that he had restored any of the original propositions.

1822).
[43] Ibid., p. 179.
[44] Ibid., p. 182.

Playfair instead goes back to the original motivation of the ancients. They were "meticulous searchers for the solutions to problems."[45] He noticed that in their cautious practice of trying to lay hold of "every variety" of a problem before they considered it solved, the ancients sometimes ran across problems which would admit of no solutions, in that the "general construction by which they were resolved would fail, in consequence of a particular relation being supposed among the quantities which were given; and it was readily perceived, that this always happened when one of the conditions prescribed was inconsistent with the rest, so that the supposition of their being united in the same subject involved a contradiction."[46] This is the kind of situation reached when one is directed to cut a given line such that the rectangle under its segments should be equal to a given space (*Meno* problem):

> It was evident that, if this space was greater than the square of half the given line, the thing required could not possibly be done; the two conditions, the one defining the magnitude of the line, and the other that of the rectangle under its segments, being then inconsistent with one another. Hence an infinity of beautiful propositions concerning the maxima and the minima of quantities, or the limits of the possible relations which quantities may stand in relationship to one another.[47]

In the *Meno* problem, I will show that it is the finding of the maxima that demonstrates the compatibility of knowledge and teachability.

Playfair noted that in more complicated cases, ambiguous or obscure constructions might result. Rather than there being definite points in the intersection of curves (loci), which would have indicated determinate solutions, or the complete lack of intersection, which would have indicated no solution whatsoever, there would instead result lines which fully coincided, which seemed to indicate an infinite number of

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[45] Ibid., p. 185.

solutions. This embarrassment of riches was soon understood as signifying that, although the number of solutions was innumerable, "they must all be related to one another, and to the things given, by a certain law, which the position of the two coinciding lines must necessarily determine."[48] Playfair concludes that, "It was not difficult afterwards to perceive, that these cases of problems formed very curious propositions, of an intermediate nature between problems and theorems, and that they admitted of being enunciated separately, in a manner peculiarly elegant and concise. It was to such propositions, so enunciated, that the ancient geometers gave the name of Porisms."[49]

If we are to finally understand what a porism precisely is, we must examine some examples at closer quarters. We must try to determine how it is that one type of problem can satisfy all of these multiple sets of conditions described by Pappus and the modern commentators. We must determine how they can satisfy Pappus' criteria of both being intermediate between problems and theorems, as well as that of being a locus problem less an hypothesis. And for further possible philosophical application, we must try to make sense of that property of porisms "concerning the conditions of a problem being involved in one another, in the Poristic, or indefinite case."[50]

Towards this effort Playfair reconstructs three problems which illustrate the nature of analysis in general and porisms in particular. The first problem shows the relationship between a completely determinate problem (one for which there is a single

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[46] Ibid., p. 187.
[47] Ibid., p. 188.
[48] Ibid., p. 189.
[49] Ibid., pp. 189-90.
[50] Ibid., p. 201.

determinable solution) and that particular case where it deteriorates into an indefinite number of solutions:

Proposition II, Problem (Fig. 7)[51]

> A triangle ABC being given, and also a point D, to draw through D a straight line DG, such that perpendiculars being drawn to it from the three angles of the triangle, viz. AE, BG, CF< the sum of the two perpendiculars on the same side of DG, shall be equal to the remaining perpendicular; or, that AE and BG together, may be equal to CF.[52]

We are to suppose what we are seeking to prove has been accomplished. We then bisect AB at H, draw HK perpendicular to DG and construct CH, to obtain the following figure (Fig. 7).

Let us hypothesize that the construction has been completed according to specifications. Then let us bisect AB at H. The two perpendiculars, AE and BG are together the double of HK since HK is the median of the trapezoid ABGE. By hypothesis, they are also equal to CF, so that CF must be also the double of HK. So also is CL the double of LH (by similar triangles LHK and CFL). Since CH is given in "position and magnitude", the point L is also therefore given. And since D is given, the line DL is necessarily given in position, which is what was to be found. L is the point one third up the bisector of side AB.

This problem as it stands is a simple locus construction problem with a determinate solution. Let us consider, though, the range of points D through which the specified line can be drawn and the collection of such lines. It is obvious from the proof there is one point through which all solutions pass. Indeed the proof is completely

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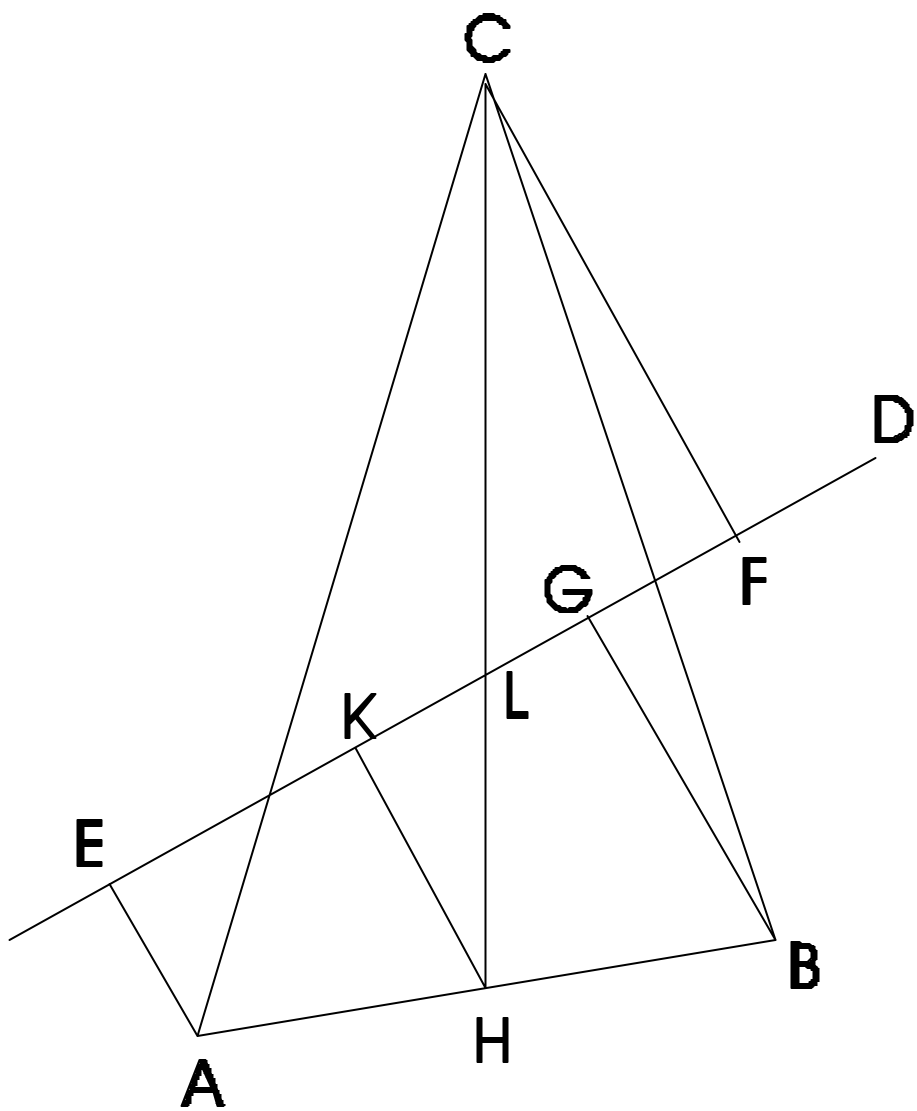
[51] In this section of Chapter Two I will number figures to correspond to the passages on Playfair.
[52] Ibid., p. 194.

Fig. 7



general and holds for any line through L. This is the poristic case of the problem, which may be stated as: "A triangle being given in position, a point in it may be found such that any straight line whatever being drawn through that point, the perpendiculars draw to this straight line from the two angles of the triangle which are on one side of it, will be together equal to the perpendicular that is drawn to the same line from the angle on the other side of it.'[53]

There are some features we should note about this problem as originally stated. First, it is a linear locus problem rather than a plane or solid locus. In general, as we mentioned before (p. 51), these can be much more complicated than the other two types.

Second, we should note how this analysis differs from that of the *Meno* problem. In that locus problem the limiting case was that of impossibility not the indefiniteness of a case with unlimited solutions.

A third interesting property of this problem is that it may be generalized further. If, instead of taking the vertices of a given triangle, we could suppose as many points on a plane as we like, then the porism would indicate that the sum of all the perpendiculars from the points on one side of the line would equal the sum of the perpendiculars from the other side of the line. And further, if instead we do not limit the given points to being in the same plane, we come up with our found point as the center of gravity of the given points.

This third feature is interesting because we see how, through the porism, one problem can be easily connected to many others. Also, the relationship between porisms and the finding of the center of gravity seems to have some importance for both Archimedes' development of the "mechanical method", as well as Descartes manipulation of the coordinate axes.

Another example will make these sets of relationships even clearer.

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[53] Ibid., p. 195.

Proposition III, Problem, Fig. 8

A circle ABC, and two points D and E, in a diameter of it being given, to find a point F in the circumference of the given circle, from which, if straight lines be drawn to the given points E and D, these straight lines shall have to one another the given ratio of V to $.[54]

As is proper for the analysis, we now assume the problem to be solved, so that F is found such that FE is to FD as the given ratio of V to $. We now construct EF to B and bisect angle EFD by line FL and angle DFB by line FM, to produce the following figure (Fig. 8).

Now, because angles EFD and DFB are bisected, the following lines are in a given ratio (by Euclid VI, 3 " If an angle of a triangle be bisected and the straight line cutting the angle cut the base also, the segments of the base will have the same ratio as the remaining sides of the triangle):

EL : LD :: EF : FD

Now this theorem is extended by Pappus, utilizing a secondary theorem that first appears in Aristotle's *Meteorologica* (III, 5, 376a3)[55], to include both the interior (EFD) and *exterior* (DFB) angles of the triangle. So that we also have:

EM : MD :: EF : FD

So that ED, EL, LD, EM, MD and therefore points L and M are given or determined.

And since angle LFD is half the angle EFD, and the angle DFM is half the angle DFB, and the two bisected angles together add up to a straight angle, the sum of angles

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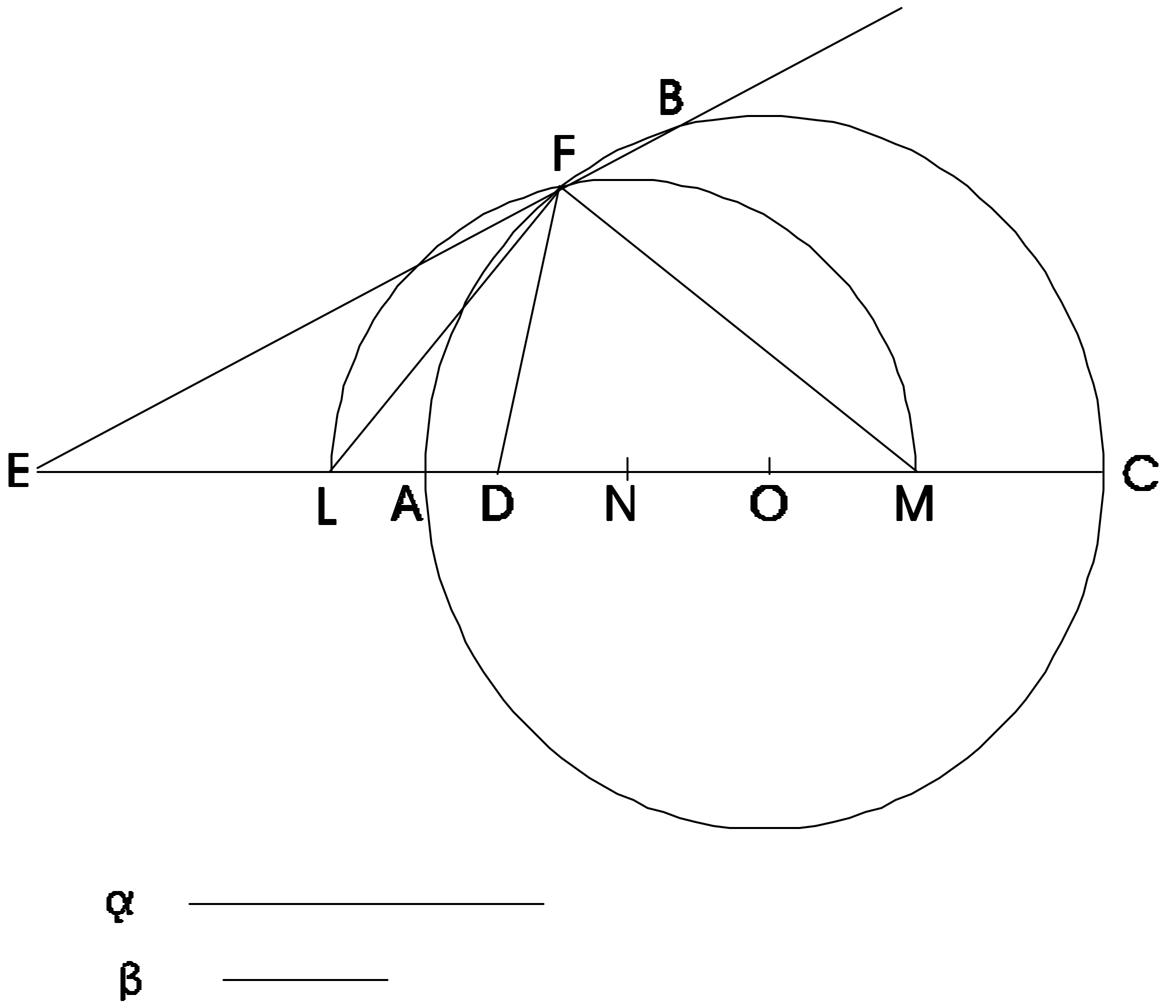
[54] Ibid., p. 196.
[55] Heath, *Elements, Vol. II*, pp. 198-99.

Fig. 8



LFD and DFM are together a right angle. Therefore point F is on the circumference of a circle whose diameter is LM, so given in position. But F is also on the circumference of the given circle ABC, and is the intersection of the two circles, and is therefore found.

The synthetic reconstruction starts by dividing ED at L such that EL is to LD as the given ratio of " to $. Then extending ED to M such that EM is to MD in the same ratio as " to $. Then bisecting LM at N, construct the circle LFM on the radius NL, with point F as the required determination.

This problem is interesting for an assortment of features which seem to confirm many assertions of the previously mentioned commentators. It is in some ways a mixture of the conditions of our previous two problems (*Meno* problem and Problem II). If the circle LFM falls either completely within or without circle ABC, the construction fails, as in the limits examined in the *Meno* problem. And when the circumference of LFM fully coincides with that of ABC, then the construction fails because there are an unlimited number of solutions and the problem is indeterminate, as with the previous problem. Like the former problem this problem is convertible into a porism. To this end we must consider under what circumstances that points L and A, as well as points M and C will coincide, bringing the two circles into coincidence (Fig. 9).

Following this supposition, we may conclude that:

EA : AD :: EC : CD :: " : $

This being the direct transformation of the previous given ratios applied to the new construction. Also by conversion we have:

EA : EC :: AD : CD

and

EA : AC :: AD : (CD - AD)[56]

Or

Since O is the center of the Circle ABC, (CD - AD) = 2 OD

Since

AO - AD = OD

CO - AD = OD

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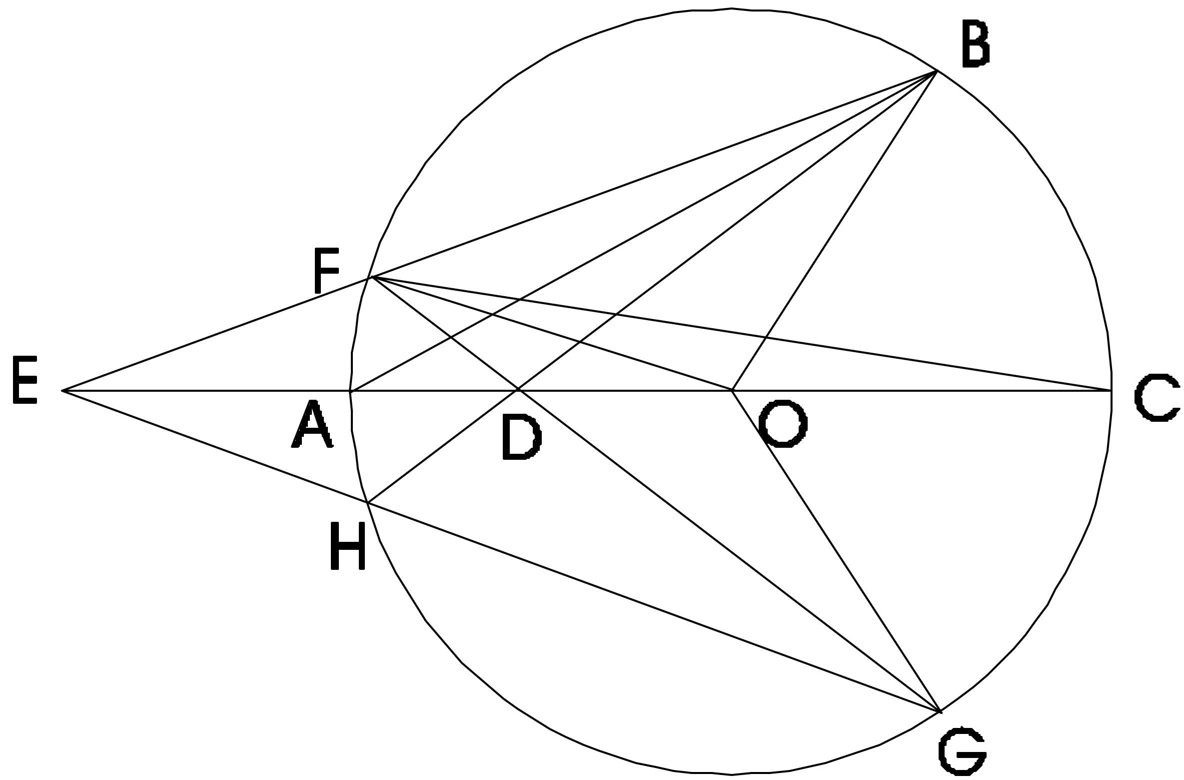
[56] Euclid, *Elements, Book V,* Proposition 5: *If a magnitude be the same multiple of a magnitude that a part*

Fig. 9



So:

(AO + CO) - 2AD = 2OD

But

AC - AD = CD

So

CD - AD = 2OD

And

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subtracted is of a part subtracted, the remainder will also be the same multiple of the remainder that the whole is of the whole.

EA : AC :: AD : 2 DO

So also:

EA : AO (or .5AC) :: AD : DO

and

EA + AO : AO :: AD + DO : DO

or

EO : AO :: AO : DO

or

rectangle EO * OD = square AO.

So, if given points D and E in respect of circle ABC, so that rectangle EO*OD is equal to the square of AO on the diameter of the circle, and also EA to AD is in the ratio of "to $, this problem admits of innumerable solutions and is indeterminate. The corresponding porism can now be stated as: "A circle ABC being given, and also a point D, a point E may be found, such, that the two lines inflected from these points to any point whatever in the circumference ABC, shall have to one another a given ratio, which is to be found (" : $,)."[57]

Although Playfair does not claim that this porism is one in Euclid, he finds it interesting for its easy conversion into multiple others, some of which were included in Euclid's work. So as another example of how easily one porism is derived from another, Playfair modifies this same problem in the following way.

Taking the circle ABC and points D and E from this last problem, we construct through D any line whatever HDB, meeting the circle at B and H (Figure 9). Then,

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[57] Ibid., p. 199.

constructing lines EB and EH, these lines cut off equal arcs of BF and HG. And arc BC is also equal to arc CG. So that if FC be drawn, then angles DFC and CFB are equal (two inscribed angles cutting off equal arcs are equal). Also if we draw OG and OB, then Angles BOC and COG will also be equal (two central angles cutting off equal arcs are equal), and therefore also their complements, angles DOB and DOG will be equal. And if AB is drawn, then it will bisect angle DBE, making it clear that angles AOF and AOH are also equal, and therefore angles FOB and HOG as well as arcs FB and HG.

Since we have previously established that if E or D be given, the other may be found, we have what Playfair claims is the last porism in Euclid's third book: "A point being given, either without or within a circle given in position, if there be drawn, any how through that point, a line cutting the circle in two points; another point may be found, such, that if two lines be drawn from it to the points, in which the line already drawn cuts the circle, these two lines will cut off from the circle equal circumferences."[58]

This proposition offers a clear illustration of the remark that the conditions of a problem are involved with one another in the porismic or indefinite case:

> Now, these conditions are all independent of each other, so that any one of them may be changed, without any change whatever in the rest. This at least is true in general; but nevertheless in one case, viz. When the given points are so related to one another, that the rectangle under their distances from the centre, is equal to the square of the radius of the circle, it follows from the foregoing analysis, that the ratio which the inflected lines are to have to one another is no longer a matter of choice, but is a necessary consequence of this disposition of the points. For if any other ratio were now assigned than that of AO to OD, or, which is the same, of EA to AD, it would easily be shown, that no lines having that ratio could be inflected from the points E and D, to any point in the circle ABC. Two of the conditions are therefore reduced into one; and hence it is that the problem is indefinite.[59]

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[58] Ibid., p. 201.

From this account, Playfair moves directly to define a porism as "A proposition affirming the possibility of finding such conditions as will render a certain problem indeterminate, or capable of innumerable solutions."[60] He believed that he could apply this definition to all of the various characteristics ascribed to the ancient poristic "without difficulty". For one, this definition left open the possibility that porisms were neither theorems nor problems, but of an "intermediate nature between both": "for they neither simply enunciate a truth to be demonstrated, nor propose a question to be resolved; but are affirmations of a truth in which the determination of an unknown quantity is involved."[61] To the degree that they do assert that some propositions may become indeterminate, they share in the nature of a theorem. To the degree that they attempt to discover those conditions by which this is brought about, they share in the nature of problems.[62]

Playfair believes that this definition can also help us to see why Pappus described a porism as a locus theorem deficient by an hypothesis:

> Now, to understand this, it must be observed, that if we take the converse of one of the propositions called Loci, and make the construction of the figure a part of the hypothesis, we have what was called by the ancients, a Local [locus] Theorem. And again, if, in enunciating this theorem, that part of the hypothesis which contains the construction be suppressed, the proposition arising from thence will also require, to the full understanding and investigation of that truth, that something should be found, viz. The circumstances in the construction, supposed to be omitted.[63]

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[59] Ibid., p. 202.
[60] Ibid.
[61] Ibid.
62 Ibid.
63 Ibid., p. 203.

In our previous problem, if we say that from the two given points D and E, two lines EF and FD are drawn to a point F, such that they are in a given ratio, then the point F is on the given circle, we have a locus problem.

If we likewise say that given circle ABC with center O and point E, with D somewhere on the line OE, such that the rectangle EO*OD is equal to the square on AO, and that any lines EF and DF, drawn to any point on the circle F, these lines will be in the given ratio, we have a locus theorem.

But then if we state that a circle ABC and a point E are given, and that we may find a point D such that any lines DF and EF to a point on the circle will all be in a given ratio to each other, then we have a porism. So that the local theorem can be changed into a porism, "by leaving out what relates to the determination of point D and the given ratio."[64] This is seeming confirmation of Pappus' claim that a porism is a locus theorem less an hypothesis.

Playfair reaffirms, however, that his does not justify the claim that all porisms may be fully convertible with locus theorems: "But though all propositions formed in this way, from the conversion of Loci, be Porisms, yet all Porisms are not formed from the conversion of Loci."[65] Playfair here concludes that he has finally settled the question about the nature of the ancient propositions called porisms.

Having settled the question as to what porisms were, Playfair continues on to develop further insights as to their utility and application. His first observation about the nature of these propositions is a clear and focused conclusion as to the kind of analysis which is best suited for poristic investigation:

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64 Ibid., p. 204.

> If the ideas which we have given of these propositions be just, it follows, that they are always to be discovered by considering the cases in which the construction of a problem fails in consequence of the lines which, by their intersection, or the points which, by their position, were to determine the magnitudes required, happening to coincide with one another. A Porism may therefore be deduced from the problem it belongs to , in the same manner that the propositions concerning the maxima and minima of quantities are deduced from the problems of which they form the limitations; and such no doubt is the most natural and most obvious analysis of which this class of propositions will admit.[66]

There are significant insights from this passage that can affect the way in which we might eventually attempt to apply the structure of mathematical analysis to philosophy, or rediscover how it was in fact done. First there is the relationship between the systematic failure of an hypothesis and the ability to systematically deduce determinate consequences from the precise details of that failure. This quality seems well suited to aid Plato in his struggles with the skeptics and relativists. In almost all of the dialogues, Plato seems to move systematically from failed hypothesis to failed hypothesis, often ending in the same. If we could better understand some methodological progression in this movement, perhaps the solutions of those same dialogues would begin to emerge.

Further, there is the interesting relationship noticed between the derived limits of an hypothesis and the determination of optimization of the solution (maxima/minima). This corresponds nicely to the development of the equilateral triangle as the paradigm or optimum measure that determines those rectilinear figures which can be inscribed in a given circle (second *Meno* problem). So while the second *Meno* problem, since it

65 Ibid.
66 Ibid., pp. 209-10.

examines the consequences of an impossibility rather than an indefiniteness, appears not to be a porism, there is a close parallel between the use of the concept of limits in both.

Playfair then asserts that this condition is not the only one that can frame poristic analysis. He speculates that it would be useful to be able to have a means by which porisms could be "found out, independently of the general solution of the problem to which it belongs."[67] In fact, he concludes that there is reason to believe that porisms may be investigated more easily than the general problem, and then turned around to help resolve that problem itself. This seems to correspond with Pappus' observation that, "a porism [is] that which is proposed with a view to the producing of the very thing proposed. So that in the previous problem, if we had first resolved the porism for the indefinite case, the solution of the construction problem for the definite solution would have followed as a limiting subset.

2.4 Summary Review

We have in this search, uncovered two distinct forms of analytic reasoning. The one, as represented in the examination from the *Meno*, seeks to find the conditions for the possibility of making some construction (*diorism).* It is the preliminary investigation to solving a certain kind of locus problem. This thinking is dioristic.

The "downward path" of analysis seems well represented by dioristic, as it helps determine the kinds of figures which can be constructed within certain loci. These proofs approximate to the logical process of determining which particulars could be included under a given universal. The more complicated local theorems involve the relationships

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67 Ibid., p. 210.

between indeterminate loci, or conic sections, and are referred to as "solid" loci. Menaechmus mastered this particular strategy at the Academy. The paradigm problem for dioristic analysis is the finding of tangencies through the analysis of maxima and minima.[68] Before him these problems were mainly dealt with via reductions to simpler problems and mechanical constructive devices.

The "upward path" of analysis appears to be that of poristic. Poristic, like dioristic, is a kind of locating of limits. The difference seems to be that, whereas dioristic limits are local optimums of specifically defined quantities, poristic limits are relations between indefinite quantities.

Another application of this kind of reasoning is that of exploring the possibility for the quadrature of a continuous curve. The ancient method of handling this problem was the "method of exhaustion of Eudoxus. Again, we are in the position of finding some invariant relation from allowing some process to go on to an indefinite or unlimited recursion. This sophisticated technique bears comparison with the calculating of limits that developed, much later, into calculus.

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[68]Mahoney contends that this procedure of differential calculus was not unique to the moderns, nor invented by them. He shows that both Viete and Fermat "discovered" the procedure in a comment of Archimedes on the symmetry of finding roots for a quadratic equation. Viete had noticed in a comment of Pappus on Archimedes Proposition 61, that the constants of an equation could reveal more than just the roots, but also the "unique and the least" parts of the curve. This clue led Fermat directly into an investigation of the nature of the symmetry axes of higher curves and the relationship between tangencies and maxima-minima (Mahoney, pp 150-7).

CHAPTER 3

THE PHILOSOPHICAL ADAPTATION OF ANALYSIS

In Chapter One we began consideration of the question, "What is philosophical analysis in Plato?" in order to explore how Plato attempts to utilize this method to defend the claims of knowledge against the relativists (Protagoras) and the skeptics (Heraclitus). It was suggested by Gulley that the long dispute over the nature of Plato's analytic method was possibly rooted in the possibility that there might really be two very different kinds of analysis - deductive (Robinson) and non-deductive (Cornford).

In Chapter Two we saw that there are indeed distinct types of mathematical analysis. One, rectilinear, seems closely aligned with what Cornford wants to portray as a non-deductive, problem-solving heuristic. Mathematically it can be best represented by the Slave Boy problem in the *Meno*. The other two, both involved with methods considered "indeterminate" by ancient techniques, seem more closely aligned with what is understood as "modern" analysis, and what Robinson would claim as "deductive".

In this chapter I will review and compare examples of two of these methods, rectilinear and diorismic, in the dialogue in which the concept of analysis is first introduced, the *Meno*. I will attempt to show the differences between the two mathematical problems introduced in the dialogue and, with the help of the mathematical relationships developed in Chapter Two, determine whether either utilizes a procedure that in any way could be called "strictly deductive".

I will in turn try to show how answering the question about the logical structure of analysis - whether it is deductive or non-deductive - concurrently helps to resolve the skeptical paradox of learning which introduces the problem of method in the dialogue. I will try to show that Plato (1) formulates the philosophical problems of the dialogue - "What is virtue?" and "Is it teachable?" - as if they were problems of mathematical analysis and then (2) attempts their solution according to the techniques of geometrical analysis.

I argue that the application of mathematical procedure goes beyond a merely "psychological" heuristic, in that it is neither random nor fortuitous. Yet it equally stops short of being a purely "mechanistic" procedure in that it requires making interpretations of partial results and reorienting them within the method. I will try to show that mathematical analysis is a "diagnostic" procedure that offers specific tools by which to resolve distinct kinds of problems. One must both know the map (mechanical) and the landmarks (psychological) and be able to put them together (systematic) to "know" the way to Larissa. Such is the nature of the systematic diagnostic that Plato presents as geometrical analysis.

In the course of the dialogue, the participants fail in all their efforts to discover the nature and teachability of virtue. I will show that, at the same time, Plato manages to show the reader how these subjects can, in some sense, be known. He is teaching us what the arguments seem to show cannot be taught. Hence the mathematical techniques of analysis appear to be ways of coming to learn and know virtue, in direct opposition to the explicit conclusions of the discussion.

I will have to demonstrate that there is a kind of knowledge (virtue) that, although it cannot be directly taught, can be approached and illuminated by indirect means. To do this I will need to show that within the mathematical demonstration there are distinctions made between what the constructions can illustrate directly and what they only show by indirect relationships.

I believe that this tension between the actual advancement of the reader and the failure of the arguments is more than just ironic play. Once we have come to realize this conflict between the logic and the mathematics of the dialogue, we are meant to become aware of the many textual hints in the dialogue which are there to help us reconstruct more adequate arguments for the nature of virtue and its relationship to teachability.

3.1.1 The Slave Boy Problem

It is somewhat controversial even to discuss Plato's first mathematical example, the Slave Boy Problem, under the guise of analysis. He gives no indication, as he does with the Second Problem, that this construction relates to any special method of the geometers. And differently, from the Second Problem, this first "rectilinear construction" would not necessitate any of the methods which we might call "modern analysis".

There are two senses, however, in which it can fairly be called a problem of analysis. As we mentioned in the glossary of Chapter One, in its widest sense, analysis just meant the "taking apart" of something. The "analysis" could be of a physical object, a mathematical construction, or the logic of an argument. In this sense any time we make and modify a construction, or ask ourselves how to solve a problem, we are doing an analysis. In this sense the dividing of the square was certainly a kind of "analysis".

But there is also a more special sense in which the Slave Boy problem could be included within analysis. All locus problems were considered by the ancients to be problems addressed through analysis. And there are three kinds of locus problems. Porisms are usually examples of the most complex loci, known as *linear* loci. Typically, they involve finding some center or middle for a "field" of random lines. Diorisms, like the Second Problem of the *Meno*, are referred to as *solid* loci, since they demand the use of a conic section to be resolved, and conic sections require the methods of "solid" geometry. Both linear and solid locus problems are considered "indeterminate" with respect to the methods of "plane" or Euclidean geometry.

The Slave Boy Problem involves the locating of the side of the Double Square. The side of the Double Square is incommensurable with the side of the given square. The side of the Double Square is a surd or root. The finding of square roots is typically accomplished through the constructing of a semicircle about a line (diameter). The diameter is then divided into two segments in such a way that the product of the two segments is the square whose root is desired. When a perpendicular from the point joining the two segments is drawn to the circle, it becomes the altitude to the hypotenuse of the included right angle - it is the mean proportion between the segments, ie. the surd of their product. This problem, as so stated, is a *plane* locus problem.

But *locating* the side of the Double Square requires neither that we find what the surd's length is nor that we know that the diagonal is incommensurable with the side. In Euclid, this kind of problem would typically be found in Book I, long before the nature of irrationals are even discussed (Book VII). The problem as it is stated to be resolved in the dialogue is not a locus problem at all. This is a typical rectilinear congruency problem. It is merely a case of breaking the given square into its parts and then constructing a further square with the given square as a part. Then the different areas are compared until the sought after relationship is uncovered.

There are some features of this construction to note relevant to the debate between Cornford and Robinson. First, it is fairly clear that according to Robinson, the Slave Boy problem would not be considered an example of analysis. It does not require any of the "reductions" of modern analysis and it is not convertible into the language of equations. He would hold, no doubt, that this is the reason that Plato does not refer to its solution as involving the "method of the geometers."

Cornford, on the other hand, would find this problem a good example of what he takes to be the "backward moving" process of discovery utilized by mathematicians. In fact the very way by which Socrates manipulates the slave boy by a systematic questioning would seem to demonstrate this "backward reasoning". Although Cornford does not specifically refer to this problem as analysis, it meets the criteria he seems to assert for such a method.

What is of interest to our discussion about the possible nature of analysis, is how the Slave Boy Problem *differs* from that of the Second Problem, a locus analysis. Towards this end it makes sense to understand the goal sought in examining the "Double Square" Problem.

Socrates uses this geometrical example to demonstrate that "learning" is a matter of "recollection." He will show that the uneducated slave can recollect geometrical knowledge which he has never been taught, in order to demonstrate that inquiry, or learning, is possible.

The exploration of the Slave Boy Problem illustrates that "new" knowledge may be "learned" or "discovered". Somehow by dividing the square up into sections and then constructing a larger square of which the original is merely a part, the slave is led to "see" or recognize that construction which represents the side of the double square. Even though he is led to this discovery, he is never directly told the answer, and his final recognition is clearly of a different level of conviction than his prior guesses. He has "learned" or recollected something he did not know he knew.

There are two features of this process we must look at further. First we must examine the claim that this process, by its very wandering nature, is not determinate at all but merely a matter of intuition and guessing.

The reason why this rectilinear problem can be said to be "determinate" rather than guesswork, has to do with the nature of part-whole constructions. Any rectilinear figure can be broken up into a finite number of triangles. As a figure with a finite, relatively limited number of elements, it can be evaluated with a finite procedure.

But there is still the question of whether the slave would ever have discovered on his own how to draw the diagonal line of the original square. There are two ways in which to see how this could have come about. One way relies on the intuitive method of "seeing" the solution. In this way, much as the account in the dialogue goes, the slave recognizes that "half" the area of the square of "double side" (16 units) will be the desired areas (eight units). He then "sees" that the way to divide the large square into a smaller square with half the area is to draw diagonal "corners" which amount to the diagonals of the original square.

But there is also another possible path to solution. When one came to a dead end after dividing and doubling "sides" of the original square, one would have to reexamine one's approach. So if dividing sides did not produce the right configuration of areas, maybe dividing "areas" is the better way to the solution. Once one looks at dividing area, which are not just the division of sides, one comes to the division by the diagonal.

Whether or not one comes to this division by "insight" or "principle", once one finds the diagonal cut, it is the principle of dividing areas to get areas which determines the proper solution. It is the recognition of this *principle*, even if only in a visual recognition, that the doubling of the area must somehow involve a dividing of areas that produces the *conviction* in the slave that he has finally located the proper relationship.

The second issue at play within the Slave Boy Problem is the nature of the knowledge "discovered". Even though what is sought in this problem is a "complete", self-sufficient knowledge - like the definition of virtue - what is finally attained is something quite different. There is no way to give a "complete" rational account of the diagonal of the square. It is an irrational "surd". What we can give is a geometrical relation that represents this surd, but not a single unitary answer for what the surd is. So that while the rectilinear construction which the Slave Boy Problem identifies, can be used to obtain new knowledge, it seems that the nature of that new knowledge can only be *relational*.

We might now ask the question, "What does this problem show us about the nature of virtue?" Socrates was looking for the definition of virtue. Meno offered him a "swarm" of definitions (72b). Socrates insisted that for a definition to be adequate, it had to be of the nature of a "unity". Does our mathematical solution provide such a unitary answer?

On the contrary, the very nature of our mathematical demonstration was to look for various relations among the variety of ways of ordering the parts. In the end, Socrates could never give a single, unitary answer as to the "measure" of the side of the Double Square. As we have seen, it is an "irrational" surd. So that while Socrates has demonstrated that learning, or recollection, can take place, it is still a mystery whether what we learn can in any way be that unity which must be knowledge, or virtue.

3.1.2 The Analysis of Virtue

In Chapter Two we examined the nature of the locus problem in the *Meno,* with an eye to recognizing how the structure and orientation of the problem could tell us something specific about the nature of geometrical reasoning. We showed there that the kind of locus construction described in the second *Meno* problem involved the reducing of a complex determinate problem down to a set of indeterminate relationships ("solid" loci), which could then be mutually solved to decide whether the construction was possible.

The inclusion of this particular type of mathematical example illuminates the character of the philosophical problem that we are meant to resolve. The problem (whether virtue is teachable) is supposed to be solved by determining the conditions for the possibility (diorism) of when one kind of problem can be successfully reduced to, or interchanged with, another. Now that we have introduced the way in which this kind of tool can be utilized to resolve certain classes of mathematical problems, we can begin to comprehend its reference to the philosophical inquiry of the *Meno*.

I will make the case that, within the inquiry of the *Meno*, this mathematical problem is directly related to the question of adequate definitions and their role in philosophical arguments. I will contend that the geometry of this problem is a model for determining the circumstances under which the antecedent (sufficient condition) and consequent (necessary condition) of a conditional can be converted. In other words, *diorisms* help us to determine when a successful analysis may be "converted" directly into a synthesis or "demonstration". By showing the strict conditions under which some analyses can be converted into deductive syllogisms, we will be elaborating the fundamental definitional grounds that hold between the concepts of knowledge, teachability and virtue. In this manner the mathematical problem of analysis will help to delineate the limits and possibilities of interaction between the philosophical concepts.

The second half of the *Meno* (86c-100b)- the part which is explicitly stated to be working with the method of hypothesis as an example of the analysis of the mathematicians - includes a series of hypotheses meant to answer the question whether or not virtue can be taught.

The hypotheses are structured and organized to follow the diorismic orientation of the mathematical example of analysis. In the geometrical problem we were to determine whether a certain rectilinear figure could be constructed along the diameter of a circle by examining the hypothesis: “If this triangle is such that, when laid along the given line it is short by a space as large as the figure laid along it, then I think that one thing follows, whereas another thing follows if this cannot be done. So I am willing to tell you on this hypothesis, about inscribing the triangle, whether it is impossible or not"(87b).

Socrates then suggests that they should do the same with virtue: "let us investigate whether it is teachable or not by means of a hypothesis, and say this: Among the things existing in the soul, of what sort is virtue, that it should be teachable or not? First, if it is another sort than knowledge, is it teachable or not, or, as we were saying, recollectable"(87b)? As with the geometrical problem we are going to examine the hypothesis of whether virtue is teachable by considering some “limiting” condition. If it meets the condition, then one thing follows, if not, then another. But we must be attentive to how Socrates maneuvers this hypothesis very carefully. It will not be completely surprising if some of his suggestions, like those to the slave, are somewhat misleading.

The first thing we must do to apply this method is to recognize just what is the hypothesis. This determination is no simple matter. There are three distinct propositional structures that have some justification to the claim of being hypotheses.

1) First, there is the complete contrarial pair: "if A something follows, if not A something else follows". Socrates directly refers to both the mathematical pair and the philosophical pair as hypotheses.

2) Or we could take the conditional statements themselves as the hypotheses: "if virtue is knowledge then it is teachable" or "if virtue is teachable then it is knowledge". These conditional propositions have the advantage of being the statements which are actually examined through most of the dialogue. And in logical parlance arguments utilizing conditionals are normally called "hypothetical" arguments.

3) But there is reason to assert that the actual hypotheses are something else. In our method we are told that we are to "assume" that which we are trying to prove, and that it is what we are assuming that is our hypothesis. In this context then it will be the antecedents of our conditionals which will be the most appropriate propositions to be called "hypotheses".

I will utilize the convention of calling the larger hypothetical set "contrarial pairs". I will refer to the individual branches of the contrarial pairs just as conditionals. And I will refer to the antecedents of the various conditionals (alt. 3) as our "hypotheses".

As in the mathematical problem, the philosophical contrarial pair is set up as a mutually exclusive set of conditionals. The first fork is put forward as a strong assertion by Socrates: “Isn’t it plain to everyone that a man is not taught anything except knowledge?(87c)” This statement can be logically restructured as “ If not knowledge, then not teachable." When examining the complete pair, we must keep in mind Socrates strong language in affirming this conditional branch as "plain to everyone."

The second fork, which Socrates offers up almost immediately is the more recognizable conditional: "If virtue is knowledge then it is teachable (87c)." The two conditionals together seem to make up the kind of pair needed to meet the conditions of our diorism: If virtue is knowledge then it is teachable, and if virtue is not knowledge then it is not teachable.

There are some features we should note about the setting up of this pair of conditionals. First we must evaluate whether or not the conditionals are really exhaustively exclusive. It is clear that one of the possibilities that this pair excludes is that there might be some knowledge which is not teachable. That this possibility is excluded becomes immediately evident in Socrates' rephrasing of the pair in terms of the contrapositive of the negative conditional - If virtue is not knowledge then it is not teachable = If teachable then knowledge:

> S: We thought it could be taught if it was knowledge? - Yes.
> S: And that it was knowledge, if it could be taught? - Quite so (98d).

This restructuring of the hypotheses makes it more clear that the condition for the exclusivity of the original two conditionals is that they together, as restructured, make up a biconditional. Or, in other words, teaching and knowledge must be interchangeable if the conditionals are to be convertible with one another. So that if we have a convertible syllogism (i.e. one with a biconditional major premise), then there *will not* be a kind of knowledge that is unteachable, for the contrarial pair will be mutually exclusive (If teachable then knowledge and if not teachable then not knowledge).

It also seems that Meno ignores the contrarial structure of the argument and instead only wants to examine one side of the biconditional: If virtue is knowledge then it is teachable. Socrates appears uncomfortable with this approach: " I suppose so. But I wonder if we were right to bind ourselves to that? (89c)" Although Socrates has led Meno to this focus on one direction of the implication, like with the Slave Boy Problem, Meno cannot recognize his error.

And latter Plato gives another indication of his concern with the status of the investigation, "I will tell you, Meno. I am not saying that it is wrong to say that virtue is teachable, if it is knowledge, but look whether it is reasonable of me to doubt whether it is knowledge? (89d)" Socrates does not seem to agree that this isolated proposition is the most fruitful way to examine this condition.

Socrates' concern for the way in which Meno is examining the question of the teachability of virtue can be made clearer if we translate that conditional into a syllogistic form:

Knowledge is teachable
Virtue is knowledge
Virtue is teachable

It is clear from this syllogism that the conditional that Meno is examining - if virtue is knowledge, then it is teachable - will not satisfy the conditions of the analytical method. In this syllogism we are assuming that virtue is knowledge in order to prove that it is teachable. In the analytical method the reverse is called for: "Now analysis is a method of taking that which is sought as though it were admitted and passing from it through its consequences in order to something which is admitted as a result of synthesis."[1] The analysis that we are to examine is the nature of a proof procedure for attempting to demonstrate that which we first have to assume.

We are again led back to the other horn of the contrarial pair. When we translate this other conditional - if virtue is teachable then it is knowledge - into its syllogistic form, we finally attain the analytical proof that is called for:

The teachable is knowledge
<u>Virtue is teachable</u>
Virtue is knowledge

In this syllogism we are correctly assuming the hypothesis that we want to verify - that virtue is teachable. If it leads to something not true, it is incorrect: "If someone then attacked your hypothesis itself, you would ignore him and would not answer until you had examined whether the consequences that follow from it agree with one another or contradict one another (*Phaedo* 101d)."

Of course there is more than a little irony in Meno's confusion. It is Socrates who wants to pursue the question "what is virtue". Meno only wants to know whether it is teachable. In the examination, however, Meno mistakenly seeks to find whether it is knowledge in order to find whether it is teachable. Socrates, on the other hand, correctly pursues the hypothesis, "whether it is teachable", even though it is not the question that he finds most interesting.

Socrates first proceeds to examine the truth of this consequence or conclusion of the analytic syllogism. He proceeds almost immediately after stating the first contrarial pair to state a Second: "If then there is anything else good that is different and separate from knowledge, virtue might well not be a kind of knowledge; but if there is nothing

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[1] Pappus, "Treasury of Analysis," *Greek Mathematical Works* (Cambridge, 1993), p.597.

good that knowledge does not encompass, we would be right to suspect that it is a kind of knowledge (87d)." This second contrarial pair, like the first, is made up of a set of mutually exclusive conditionals. This pair is the examination of the consequence: virtue is knowledge.

The second contrarial pair is also significant for the way in which its two "horns" are put together. The contrary antecedents of both hypotheses are “exclusion” propositions themselves - “anything else” and “nothing else”. Something involving the relationship between inclusion and exclusion is intrinsically involved in determining which hypothesis succeeds.

It is this key idea, that of the relationship between good definitions and the nature of inclusion, whether within a "class" or a "species" or a "category", which calls up a picture of our geometrical problem. Our mathematical problem was to determine the conditions of when one kind of geometrical figure (a triangular area equal to a given rectangle) could be inscribed or included within another kind of figure (given circle). The problem involves the explicit examination of the conditions for this possibility of an inclusion. We must now turn to our geometrical construction to clarify better just how our two exclusively contrary pairs of conditionals are to be understood.

Before one can construct a relationship between virtue, knowledge and teachability, one must be able to identify which in turn has the wider scope. This clarification of scope is the means by which the definitions can be made more precise. In the examination of the second contrarial pair, the scope of knowledge is seen to completely contain that of virtue, “but if there is nothing good that knowledge does not encompass, we would be right to suspect that it[virtue] is a kind of knowledge (*Meno*

87e).” This explicit reference to one idea “containing” another recalls the locus problem that is the model of the discussion.

If we are to utilize the geometrical problem of analysis to clarify further this problem of conceptual comparison and definitions, we must accomplish four ends. First, we must illustrate *why* the mathematical problem works. We must show what it is that the mathematical problem demonstrates about the nature of analysis.

Second, we must begin to develop how the mathematical figures can be put in determinate correspondence with the definitional relationships of the philosophical discussion. In this endeavor, we must move beyond a merely metaphorical comparison and make an effort to demonstrate the relational conditions that dictate one particular interpretation of the construction over another.

Third, we must make some explicit prediction as to what the mathematical problem illustrates about the conditions we need to meet in the philosophical problem. How does the mathematical problem actually show us what we need to accomplish in the philosophical inquiry. And lastly, we must determine whether or not these conditions are ever met in the dialogue.

In the geometric analysis, we are to determine whether the area of a given rectilinear figure could be inscribed as a triangular area in a given circle. We first assumed that this could be done (as an isosceles triangle). We then showed that the constructed isosceles triangle was equal in area to a rectangle constructed on the diameter such that the rectangle on the remaining portion of the diameter was similar to the one first constructed. This characteristic is the equivalent of showing that the "half base" of the isosceles triangle is the mean proportional between the segments of the diameter (because it is the altitude of a triangle inscribed in a semicircle).

To transform this analysis into a synthesis, we must exchange the minor premise (the inscription of the triangle) and the conclusion (the equating of the given figure to the constructed rectangle) and show that the major premise is convertible. The major premise states that the "half base" of the inscribed triangle determines two similar rectangles. The converse of this condition is also true - that the similarity of the two rectangles on the diameter determines the inscribability of the equivalent triangle. So we have successfully reduced our original problem - whether a triangle equal to a given rectangle could be inscribed in a given circle - to whether that same area could be constructed as a rectangle on the diameter of that same circle, such that "it is deficient by a figure similar to the very figure which is applied (87a)." It is this condition of the convertibility of the relationship between the rectangular similarity and the inscribability that allows the convertibility of the analysis into a synthesis.

This condition of the reduced construction - the similarity of the rectangles - amounts to establishing that one pair of opposed vertices (A, E) will be situated on the circumference of the circle. In this way the side of the applied rectangle in contact with the circle will always determine the vertex of an inscribed right angle, guaranteeing the required similarity, since the altitude to the hypotenuse of a right triangle is the mean proportion between the sections of the hypotenuse. This condition of a second rectangular vertex lying on the circle may be represented as an hyperbola intersecting the circle. The hyperbola shows us that for sufficiently small rectangular areas, there are *two* points of intersection or distinct isosceles triangles. Only in the case where the hyperbolic curve is tangent to the circle will there be only a single solution, and in this case it is the maximum inscribeable area - the equilateral triangle.

This hyperbolic relationship with the circle, as a conic section, is a solid locus problem and therefore involves the same analysis that solves the finding of two mean proportions between two given values and resolves the famous Double Cube problem (see Chapter Two). This relationship between the Double Square and the Double Cube has not stirred much notice among the commentators, but must have at least some metaphorical significance to Plato. This relationship between the two *Meno* problems, as differentiated only by a degree of *dunamus* or power is interesting for our present study for two reasons.

First, this relationship between the dimensionality or "power" of the two *Meno* problems is reconfirmed by the history of mathematical problem solving at the academy. We showed earlier (Chapter 2) that the Double Cube Problem was the pardigmatic case for solving solid locus problems. So that the two problems in the *Meno* are each respectively examples of using plane and solid loci problems for finding the double of plane and solid figures.

Second, there is apparently a close relationship between the two *Meno* problems and the mathematical analysis that takes place in the dialogues the *Theaetetus* and the *Statesman*, specifically concerning levels of *dunamus*. The mathematical example of the *Theaetetus* (147d), like the Slave Boy Problem, is concerned with a mathematical expression for the unity of knowledge. Somehow Theaetetus' scheme has demonstrated a geometrical way to see all of the surds as a unity. Campbell and Burnyeat have pointed out that the *Theaetetus* reference to *dunamus* anticipates a passage referring to the mathematics of solids at *Statesman* 527c-528e[2]. In a semi-comic passage on how to cut

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[2] Malcolm Brown, "Pappus, Plato and the Harmonic Mean", *Phronesis* (1975),20:173-184.

the herd of tame animals, Socrates goes into a geometrical digression on dividing by diameters, diameters of powers and diameters of diameters. The references to the "*dunamus* of two feet" and the "*dunamus* of twice two feet" corresponds precisely to the way in which mathematicians referred to finding the two means in the double cube problem.[3] And, of course, this relationship of "power" or *dunamus* between these two dialogues is well anticipated by Theaetetus' comment: "And there is another distinction of the same sort with regard to solids (*Theaetetus* 148b)." The *Statesman* is a dialogue concerned with the problem whether we could ever "duplicate" the wisdom of the original law giver. He who can attain "divine dispensation" is the statesman "who can create another like himself (Double Cube) (*Statesman*)." Such a statesman, or mathematician, surely "would be just like that *solid* reality among shadows (*Meno*)."[4]

In order for this mathematical analysis to have a determinate effect on our philosophical problem, there would need to be some concrete conceptual relationship that is reflected in our geometrical construction. In our second contrarial pair, in examining whether virtue is knowledge, we were faced with a very similar kind of determination as in our construction. Socrates got Meno to agree that there are no acts of virtue unaccompanied by knowledge. Are we then justified in just identifying knowledge and virtue? There is nothing in the discussion that would preclude this assumption, but a closer examination of our geometrical model shows that this conclusion is rash (Fig. 3.1).

The value of the construction in helping us determine whether knowledge and virtue are interchangeable is "visible" before our eyes. Any triangle equal to the

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[3] Ibid.
[4] Tr. Guthrie.

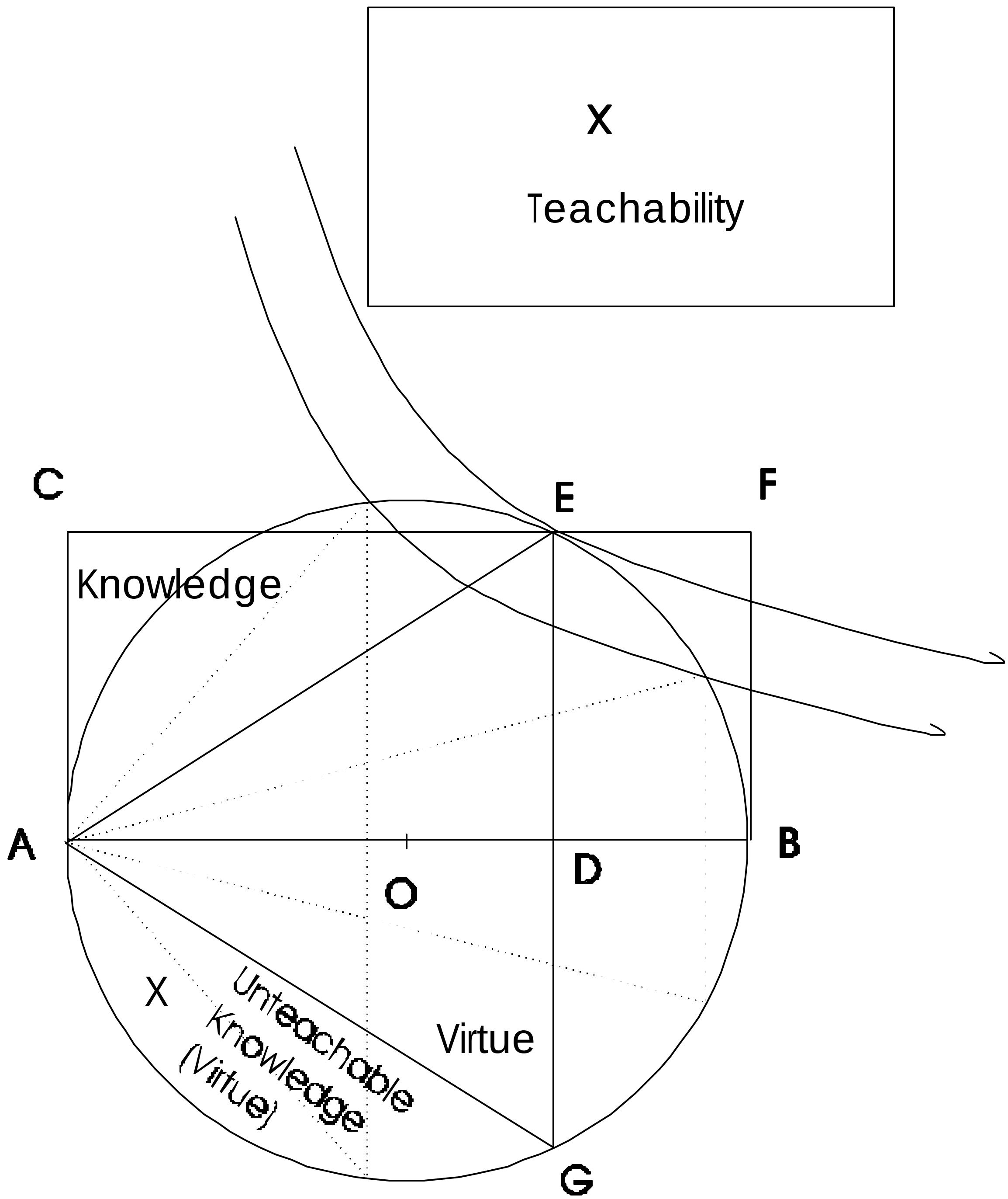
X
Teachability
C
E
F
Knowledge
A
O
D
B
X
Unteachable
Knowledge
(Virtue)
Virtue
G

Fig. 3.1

rectangle so constructed so that the remainder rectangle is similar, can be "included" within the circle. But it is clearly evident that none of these triangles is *equivalent* to the circle. The relationship of inscribability, like that between knowledge and virtue is asymmetric. The triangle may be included within the circle, *but not vice versa*.

The original question to be examined, "Is virtue teachable", is represented by the relationship between the given rectangular figure and the possibility of inscribing an equivalent isosceles triangle in the given circle. We have reduced this original hypothesis to the alternative condition of whether the given area can be constructed as a rectangle on the diameter of the given circle such that a rectangle on the remainder of the diameter is similar to the constructed rectangle. We have also noted that this problem is represented by the locus of a hyperbola intersecting the given circle.

In order to determine that this mathematical construction has any direct relevance to our philosophical problem we will have to show that it precisely signifies the same kinds of inter-relationships as we can sort out between the philosophical propositions. The hypothesis that "virtue is teachable" corresponds in kind to that of whether we can inscribe our area as a triangle. They are both hypotheses that we are to test.

In our analytical syllogism we then needed to prove the conclusion that "virtue is knowledge". In our examination of the second contrarial pair, we came to the conclusion that there could be nothing which is Good which is not accompanied by knowledge: “since all qualities of the soul are in themselves neither beneficial nor harmful, but accompanied by wisdom or folly they become harmful or beneficial (88b).” Or in other words, virtue is always contained by knowledge, but knowledge and virtue are not necessarily interchangeable. It has not been well argued by the commentators that this relationship is not one of identity. Plato's argument only demonstrates that virtue must be included within knowledge, *but not vice versa*.

And it is the geometrical construction which lays this asymmetry before our eyes. The various inscribed isosceles triangles (virtuous activities) are all contained within the circle (Divine Knowledge), but human virtue - that within a human soul - is never fully commensurable with the Divine Circle.

This "translation" from the mathematical construction to the philosophical analysis is reminiscent of the symbolism of the *Timaeus*. Circularity is the geometry reserved for the Heavens. The circular is considered most Divine because it represents both a unity without parts and a perfection of balance and symmetry. Rectilinear figures, on the other hand, are archetypal of the material, the finite and the mortal. They are made up of a limited number of parts. All the wholes may be completely broken down into elemental triangles.

From our construction, we may conclude that while knowledge, as circular, has a kind of Divine Nature, virtue "within the human soul" is of the nature of a mortal activity, as portrayed by the perfectly balanced equilateral triangle. The outer circle represents the Divine Ideas - Knowledge and the Good. The equilateral triangle, represents the optimum capacity of human action. Its rectilinear nature makes it appropriate for representing the human condition of becoming. Its paradigmatic significance as a maximum, makes it ideal for signifying the instantiation of the Ideas in human knowing and virtuous action. Only an "ideal" with some kinship to the perfect could possibly represent the Divine as captured within the human[5].

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[5] This interpretation of the application of the geometric construction to the philosophical relationships is

It is interesting to note exactly how the equilateral triangle, as representing the best of human possibility, can be said to “partake” of the divine (the circle). For one, the triangle is “limited” by the circle. This one condition of the relationship between Form and participant refers back to the example of form as “shape” or “boundary”. Socrates' first example of what a form might be was that of the limiting nature of the shape of a solid (*Meno* 76a). But the equilateral triangle may also be said to “imitate” the circle. It is the only triangle for which all of the parts are equal, and it has optimum symmetry. Although the circle does not have parts in the traditional sense, there is a way in which it can be understood as an equilateral polygon with an unlimited number of sides (Eudoxus)[6].

The fact that the equilateral triangle both is limited by the circle and imitates it can help clarify how we must classify and relate the various definitions. Isn't virtue also a Divine Idea? To the degree that virtuous activity imitates the Good, the model for virtue is Divine. The Divine Model of virtue may be the same circle as that of knowledge (a constant theme of the early dialogues). In our diagram, as the equilateral triangle, virtue seems to capture that which is both mortal (by rectilinearity) and Divine (by imitation) in the human soul.

To finish our analysis we must now consider the relationship between teachability and knowledge. In order to convert our analytic syllogism into a synthesis it is necessary

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strongly confirmed in another dialogue. In considering whether the man who understands justice itself will be able to recognize the more profane version, Socrates ponders whether he be, "sufficiently versed in science if he knows the definition of the circle and of the divine sphere itself" but cannot recognize that justice which is human (*Philebus 62b)*. This geometry of the *Meno* has something to do with determining the relationship between divine and "mixed" knowledge.

[6] Heath, p. 327.

that the major premise of our analytic syllogism - all that is teachable is knowledge - "convert" into the major premise of our synthetic syllogism -all knowledge is teachable.

We should already be cautious of the possibility of this conversion for both philosophical and mathematical reasons. The second major argument in this part of the dialogue, the argument that showed how statesmen could not teach their children virtue, is broached at a point in the analysis that indicates that it is meant to test the convertibility of knowledge and the teachable. That statesmen could not teach their offspring was meant to defeat the conditional that if virtue is knowledge, then it is teachable. If there are no teachers, virtue cannot be knowledge. It is ironic that the conversants use virtue to make the case that there is some knowledge that is unteachable. In our construction, this knowledge, as a kind of virtue, would be represented by that divine area that lies outside the equilateral triangle.

But as soon as we recognize the fact that all knowledge will not "fit" within the scope of the teachable, we also recognize, from our construction, that the first uncontested version of the premise, that all the teachable was knowledge, is itself problematic. What about the "remainder" rectangle? As a rectilinear figure it fits our condition that it is something teachable, but as "outside" the conditions of our construction, it cannot be counted as knowledge. So there is apparently not only knowledge that is unteachable -Divine Knowledge, but also that which is teachable that is not knowledge - mere practice or artless technique.

We are forced to recognize at this point that Meno and Socrates have each been discussing two distinct and incompatible ideas of the "teachable". For Meno, as a student of Gorgias, learning and teaching just are the memorizing of the parts of rhetoric. There is no difference for Meno between the teachable and knowledge. They are both just a matter of learning the pieces or parts of something like virtue. They are the artless practice or *empeiria*.[7]

For Socrates, although the teachable and knowledge are not the same, the teachable is convertible with knowledge. Teachability is represented by the rectangular figure to be inscribed on the diameter. There are two conditions for the teachable to be truly *methodos*. First it must be a techne. Human technes, as finite activities with distinct steps, can be well pictured by the given rectilinear figure.

But it is also a frequent theme in the early dialogues that to be "knowable" a true art or techne must also be "limited" or unified by some Good or end. This is the role of the circle. It makes us distinguish between two kinds or levels of teachable activities. It establishes that not all teachable activities are knowable, but only those so limited by a purpose. This distinction correlates to the "directness" of what we can teach. That which we can teach "directly" is knowable as "method".

This recognition of the difference between that which is teachable with knowledge, or method, and that which is teachable by "mindless practice", or *empeiria*, helps focus our interpretation of the construction. The "remainder" rectangle is that teaching that is outside knowledge, and therefore, "unknowable".

But is it truly unknowable? The applied rectangle represents that knowledge which is directly teachable as *methodos*. The remainder rectangle would seem to represent that knowledge which is beyond the directly teachable. Does this make it beyond human knowing? By its determinate similarity to the rectangle of teachable knowledge, it seems to defy the label of total unteachability. Somehow we have a

[7] There is a longer account of the nature of this "artless technique" in the *Phaedrus 265d-274c*.

determinate or "functional" relationship for this "unteachable" by analogy or similarity with that which was directly teachable. Somehow that which is beyond human knowing, as being directly teachable, is yet accessible indirectly by a relationship: "we were saying that people get the idea of what is likely through its similarity to the truth. And we just explained that in every case the person who knows the truth knows best how to determine similarities (*Phaedrus273d).*" So while empirical learning may not be knowledge proper, it may yet be determinately related to knowledge. It is not the true, but only the "likely" or probable (*273d).* Our ignorance is somehow "measurable".

The remainder rectangle is the "true opinion" that can guide virtue as accurately as knowledge. However, like the untied statue of Daedalus, this empirical learning runs off as not being anchored.

This relationship between empirical learning and knowledge proper is re-emphasized in the consideration of the "road" (*hodos)* to Larissa. The person who has been to Larissa can recognize the signposts, but does not truly know where Larissa is until he translates those signposts onto some map. Only then will his right opinion be anchored in knowledge.

The conclusion reached by the interlocutors as to why virtue was unteachable is that for the statesmen, virtue must have been like the empirical learning of the remainder rectangle. This "true opinion," not being "tied down" knowledge, was not teachable.

But our construction calls this conclusion into question for two reasons. First, if the virtue of the statesmen just was this kind of empirical learning, by the example of Meno's relationship to Gorgias, we know that such artless technique can, in some sense, be taught.

Instead it seems that our construction leads us to look for the "unteachable virtue" in that area which lies outside the triangles and within the circle. This "incommensurable" area would seem unteachable even though it would need to be known by those who possessed virtue. This area represents that knowledge which is Divine.

Does this mean that our analysis may yet be convertible into a synthetic syllogism? Can we convert the analytic major premise "all that is teachable is knowledge" into the synthetic major premise "all knowledge is teachable"? Not without distinguishing carefully between the activities of direct (*methodos*) and indirect teaching (*empeiria*).

When we examine more closely the nature of the "remainder" rectangle, some interesting characteristics emerge. The Remainder Rectangle is every bit as much "within" the circle as the Constructed Rectangle. In other words, even though this rectangle is not designated as teachable by method - because it does not imitate the Divine - it, nonetheless, fulfills some of the requirement requirements for "knowability." It is contained within the Divine Circle, even though it is only a "copy of a copy."

This added characteristic also means that the combination between the Constructed Rectangle and the Remainder Rectangle, together make up a closer approximation to the circle as a whole. In other words, the combined areas of both the Constructed Rectangle and its similar remainder comes very close to "rectifying" the area of the circle. When this rectangular area is at a maximum, we can conclude that the knowledge unreachable by direct means (Divine Knowledge) is "coverable" by indirect relationships.

This discovery about how the "combined" rectangles closely rectify the circle, leads us to a further uncovering. The greatest possible combined rectangle is that given when each component rectangle is a "half square." When the rectangles of direct knowledge (method) and indirect knowledge (*empeiria*) are exactly equal, then their combination becomes a square, the quadrilateral closest in area to the circle.

This move from the equilateral triangle to the square as the optimum inscribed figure also points to another level of resolution for the problem of the knowledge of virtue (Divine). When we have a regular polygon inscribed in the circle and then increase the number of sides, the successive polygons progressively improve in their rectification the circle. Eudoxus initiated this "proof by exhaustion" by starting with an equilateral triangle and then halving each side making a hexagon.[8] He then continued and made the mathematical induction that this process would eventually capture the area of the circle. This rectification problem represents the Promethian promise that humans can aspire to Divine Knowledge within the possibility of a never-ending task (Syssiphis).

At this point it makes some sense to review what light the analysis of the geometrical figures has shed on our philosophical problem of the *Meno*. First, at a very simple level, the diagram helps us to visualize the basic logical relationship of inclusion. That which is represented by the isosceles triangles (human virtue) is included within that which is represented by the circle (Knowledge), but not vice versa. We have an *asymmetric* logical relationship.

Second, the role of *paradigm*, or ideal example, in the Platonic theory of knowledge is illustrated in an interesting way by the special isosceles triangle, the equilateral. Diorisms are fundamentally involved with *solid* locus problems. These are problems that are resolved by the determining of a limit - a maximum or a minimum. The

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[8] Heath, p.327.

equilateral triangle represents the limit of the relationship between the hyperbolic function (area of rectangle) and the circular function. And *qualitatively* the equilateral triangle is an optimum imitation of the Divine itself. It is perfect within its constraints as a rectilinear figure. It uniquely occupies that space between the Divine and the mortal. And perfect images of reality are *convertible* with that reality.

Thirdly, the diagram forces us to confront the possibility of three kinds of knowledge - Divine, *methodos,* and *empeiria*. Divine knowledge, as represented by the perfect circle, cannot be completely grasped by humans. Divine Knowledge is the only knowledge that is *directly* attainable through *nous.* We might be able to comprehend its abstract outline, but such knowledge, for humans will always seem "empty".

Indirect human knowledge accessible by method is that represented by the isosceles triangles. This kind of indirect knowledge will always be *incomplete,* although it is in the image of real knowledge. At its maximum, the equilateral triangle, it may be able to *imitate* the divine knowledge, and therefore be knowledge in the truest sense.

That practice or empirical "knowledge" represented by the Remainder Rectangle can only be referred to as knowledge in a very loose way. It is doubly removed from real knowledge, since it is an imitation of incomplete knowledge. However, it is both similar to methodical knowledge and remains within the bounds of the Divine Circle. Even though what it is similar to is itself only an image of knowledge, somehow this apparent knowledge can be utilized to increase our overall grasp of true knowledge. Somehow using the shadows in complement to the light actually helps us "see" better.

As interesting as these mathematical relationships in our construction have become, they are of no use if we cannot directly utilize them to produce corresponding propositions that then can be used in some *independent* argument. We must show that our interpretation of the geometrical construction not only meets the requirements set out by our original logical conditions, but also has guided us to some fruitful resolution of the original problem - the demonstration that virtue is teachable.

It has been my contention that Plato utilizes these constructions in a *negative* way to limit and refine the relationships between concepts. The relationship between the Constructed Rectangle and the Remainder Rectangle forced us to distinguish between two kinds of teachability and their correspondence to two kinds of knowledge - method (*methodos*) as indirect and incomplete knowledge and practice (*empeiria*) as indirect and apparent or "likely" knowledge. The difference in area between the equilateral triangle and the circle led us to distinguish between Divine Knowledge (circle) and that which seemed to share in the mortal and the eternal (equilateral triangle), as *human* knowing and virtue.

So we now have two kinds of teachability (method and practice) and three kinds of knowledge (Divine, incomplete and apparent). We also have established two conditions for the concept of human virtue. First it must be, like knowledge, contained within the circle. But equally it must be "similar" to knowledge in that its divinity is derived from its regularity.

At this point we must make some further distinctions. We have been referring to similarity in two diverse fashions. The Remainder Rectangle was similar to the Constructed Rectangle by being exactly the same shape. This similarity might be called similarity "within a genus.". We have also claimed that all the regular polygons are in some substantial way "similar" to the perfection of the circle. This similarity can be called a similarity "between genera", and is based more on an analogy between forms rather than as the identity of form.

There is an interesting relationship between these two kinds of similarity and the model of constructing musical scales. In the division that proceeds in the Eudoxian rectification, we cut each side in half. This is parallel to the creating of new octaves and is like the dialectical procedure of dividing down the middle. The construction of the Remainder Rectangle, by being within the same species as the Constructed Rectangle, is more like the intervals within the octave. These internal musical intervals are based on the "similarity" between intervals rather than that of an equal cut. The octave cuts are similar to the dialectical activity of "cutting down the middle" while cuts within the species are more like "cutting at the joints".

We must realize by this distinction between kinds of imitation that although both kinds of similar figures are in some ways "similar" to each other, they are by no means simply interchangeable. Each kind of similarity represents a different kind of "probable" knowledge. They are two distinct kinds of *enthymemes*. The similarity "within a genus" is that knowledge which is probable due to its nature as "apparent". Like the shadows in the Cave, its status as probable is based on the degree to which it is not true knowledge but knowledge of appearances. Its similarity is truer to its object, but the object it imitates is human knowing.

The area between the triangle and the circle represents potentially "real"knowledge, but is "unclear" as unknowable to man. The similarity between the equilateral triangle and the circle, as one between genera is more equivocal: it approaches an imitation of the Divine. It is probable, not as apparent knowledge, but as inadequate knowledge. So that even though I can "carry" the one similar area to cover the other, it is not clear that these very different kinds of "probables" are interchangeable in use?

In working out the compatibility between the two probable kinds of knowledge, the Cave and the Divided Line are of some aid. When the Cave is compared to the example of the Sun, the shadows represent a "lack" of knowledge. Appearances are the opposite of knowledge. But when the Cave and Sun are put within the continuum of the Divided Line, another set of relationships emerges. The Shadows really are a 'kind of knowledge' on the Divided Line. They represent a knowledge that in some way 'fills out' the blindness we are left with when we attempt to look directly at the Forms themselves.

The reason why it is necessary to be able to make this comparison between incomplete and apparent kinds of knowledge is that only one of them is "convertible" with knowledge. Only those similarities between genera, or copies of the Divine itself (equilateral triangle or square) can serve as the basis of allowing a property or essence to be interchangeable with its species. So our ability to translate indirectly apparent knowledge into such inadequate, but paradigmatic knowledge, determines the possibility of major premise convertibility - All men are rational animals/all rational animals are men. And determining this kind of interchangeability of definitional limits is what locus constructions help us to determine.

We are now in position to analyze the arguments on their own terms. There are really three distinct and parallel arguments, since both interlocutors are talking in almost completely disjunct vocabularies. There is Meno's argument for convertibility of teachability and knowledge. This argument ins based on the assumption that both teachability and knowledge are *empeiria*. Socrates' explicit argument, on the other hand, is based on the definition of knowledge as Divine and teachability as method. Although this argument cannot demonstrate convertibility, it raises the possibility of a convertibility with another kind of knowledge (paradigmatic)

Meno's argument is the easiest to dispatch. In attempting to convert our analysis using Meno's definitions, there is no problem at all converting our major proposition. For Meno teachability, as practice, and knowledge just are the same thing. The problem comes for Meno's argument in validating the analytical conclusion - that virtue is knowledge. The second argument in this part of the dialogue, that which showed that virtue could not be knowledge because there were no teachers of it, is circular in using the hypothesis to be proved (virtue is teachable) in order to prove the conclusion (virtue is knowledge). It is nonetheless sufficient to show that, under Meno's definitions, virtue is unteachable.

Second there are the definitions with which Socrates begins his discussion. Under these conditions we find that the teachability of virtue is also a problem. The problem does not come up with the analytic conclusion - that virtue is knowledge - that was satisfactorily proven in the argument that established the triangle within the circle. The problem comes in with the convertibility of the major premise - that the teachable is knowledge. Although Socrates accepts that the teachable, as *methodos*, is knowledge, he cannot establish by his original use of terms that all knowledge is teachable. Therefore the conversion fails for him also.

There emerges, however, a third way to interpret this argument from the intersection of these two parallel conversations. The resolution of the conflict in how both men used the term teachability resulted in us making a distinction between method and practice, as well as between direct and indirect knowledge. With these more refined definitions we can proceed to a Platonic view of the arguments which opens some new possibilities.

Since Combined Knowledge (direct and indirect) can be made to "rectify" the circle when it is made "similar" to the circle (like virtue) and then exhaustively divided, we can now convert the teachable (method and practice) with knowledge (as Divine = direct + indirect). Also, since the combination of direct and indirect knowledge can be constructed as a regular polygon and be "similar" to the given circle, virtue will always be within knowledge. So we can conclude that the hypothesis - that virtue is teachable, both directly and indirectly - is valid.

We have some further problems. Although we have demonstrated *that* the hypothesis "virtue is teachable" is valid under the proper interpretation of terms, we have not determined *how* it could be so taught. One of the conditions of our proof procedure was that those with virtue and those with knowledge had some "knowledge" about the Divine that guides and contains these ideas. But the Divine, as circular, does not seem to be itself *finitely* teachable, either directly or indirectly. So how can the statesman who knows the Good teach the Good?

Here we have to translate between the two senses of similarity. The two rectangles shared a similarity within a genus. This is a structural similarity. The regular polygons and the circle share a similarity between genera. It is a kind of functional similarity. There is a frequent theme in the early dialogues that by "knowing the Good" one "is" the Good, and both these conditions are intrinsically tied to "doing the Good."

This model of imitative activity as participation captures the effectiveness of the regular polygon as paradigm. The good statesman cannot directly teach his son the nature of the Good. It is unteachable by any method. The good statesman may, however, stand as the perfect instantiation of the Good and Virtue. In this way, as a paradigm or ideal, the statesman has the ability to teach the Divine, although as a moral exemplar, he can never determine just who might follow his way including whether or not they be his sons.

How can we know that this resolution of the problem is in any way more correct than that reached in the discussion? The conclusion of the discussion, that virtue is unteachable because it is merely an empirical practice goes directly against our earlier conclusion that virtue was a kind of knowledge. Our construction helped us to recognize the difference between the "teachable that was unknowable" (remainder rectangle) and the "unteachable that was knowable" (area outside the equilateral triangle). The conclusion that we reached from the consideration of all the parts of our diagram in relationship to the meaning of the picture as a whole, was that virtue was unteachable "directly" because it relied on knowledge of the divine (circle) which was outside the teachable (equilateral triangle). This result is compatible with the conclusions of both other arguments. It confirms that virtue is a kind of knowledge while also demonstrating why it cannot be "directly" taught. It is a further lesson from our construction that virtue may be teachable by indirect means, and our working out this problem seems to have been such a *methodos*.

The rectilinear figure to be inscribed as a triangle is the specific action to be judged or measured. It can be less than the equilateral triangle and still be included as that which is, to some degree, virtuous. The very fact that we can follow such a construction and determinately compare particular acts to a paradigm, as rectilinear figures are compared to the equilateral figure, is itself the demonstration of not only *that* virtue is teachable, but *how*.

So what can our mathematical construction say about the nature of virtue? Unfortunately, at one level - the Divine as the incommensurable - mathematical constructions are mute. Our construction can point us towards the ways in which human knowledge and virtue, as represented by the paradigm equilateral triangle, are both limited by and imitative of the Divine Ideas expressed by the circle. But Knowledge Itself and Virtue Itself lie in those incommensurable sectors between the human ideal and the Divine.

Just as in the Slave Boy Problem, we are led to relationships that can never be completely articulated in a "rational" expression - in the one case by the irrationality of the diagonal, in the other by the "squaring" of the circle. But different from the Slave Boy construction, we do not experience the undeniable vision of the truth in the Second Problem. The construction has been utilized in a mostly "negative" fashion. It has shown us the limits of our propositional interpretations. There has been some positive comparisons between similar figures, but this is not the same as "seeing" the resolution as in the Double Square. The construction in this problem has not "amplified" our knowledge, but merely clarified how we can limit our definitions. The work of the proof procedure was done by the propositional rules that were implemented on the propositions from the conditions of the construction. But in the very act of recognizing the specific limits of our knowledge from our constructions, we have expanded those same limits.

But our diorism was supposed to help us determine the conditions for the possibility of the hypothesis - that virtue is teachable. Those conditions should pick out particular actions by which that human action which is paradigmatic of virtue can be recognized. Otherwise, we have not progressed beyond the skeptics' claim that universal truths just are unknowable to humans (*Parmenides* 134e).

Our diorism has already informed us that human paradigmatically virtuous action is convertible with virtue itself. But more than this, our diorism has helped us find out what the conditions for the possibility of that paradigmatic actions was: Our two rectangles, the constructed Rectangle and the Remainder Rectangle, must be "similar." What this means is that the conditions for knowing that an actions is paradigmatic of the Divine, is that the two ways of knowing that action, *methodos* or knowledge of principles, and *empeiria*, or experiential knowledge, must in some definite way *correspond.* And the criterion of that correspondence is that there be a common *mean* between the two rectangles (kinds of indirect knowledge). The way in which we recognize the paradigmatically Divine, is through finding the mean between science and experience.

There is one further support that our construction lends our analysis in the Second Problem. Even though the analysis and syntheses were rule-driven and independent of the construction, there is still a second level of confirmation. The syllogistic results of the analysis and synthesis, although independently derived, can still be read back into the construction. This secondary comparison amounts to a strong sense of *consilience* with the syllogistic operations. Our premises and conclusions can be independently compared to our construction to confirm the their syllogistic connections.

The paradox of learning relies upon the nominalist presumption that there are no middle terms between the Divine and the human. The Divine is immutable and the human is in total flux and the two kinds are completely incompatible. It is built on the precarious logic that the conditions of the two kinds of "knowledge", human and Divine, as universal contraries are *exhaustive*, and therefore *mutually exclusive.* This conviction leads to two particular claims about the limits of knowledge. The first claim is that knowledge about the world of becoming can never be a "necessary and universal" knowledge. And the second holds that the "universal and necessary" realm of the Forms can never apply to human actions in the world of becoming. The logic of the *Meno* attempts to show how the logical fallacy (mutual exclusivity of negated contraries) is tied to an epistemic error (exhaustivity).

The first analysis of the *Meno*, the Slave Boy Problem, attempts to counter the first skeptical claim - that there can be no determinate knowledge "learned" about the world. The Theory of Recollection, as illustrated in the Double Square construction, attempts to show that a learning is possible that goes beyond the merely probable. Here, learning is of the "factual" kind, because it is specifically the learning about the characteristics of a figure. Yet the kind of "truth" which is demonstrated by the absolute conviction of the slave, through his rising to the principle represented by the constructed diagonal, demonstrate that some kinds of factual knowledge can be both universal and necessary. Even though the slave could not state the principle or even state the measure of the diagonal, his "seeing" the truth was an absolute demonstration of the universal necessity of the figurative relationship.

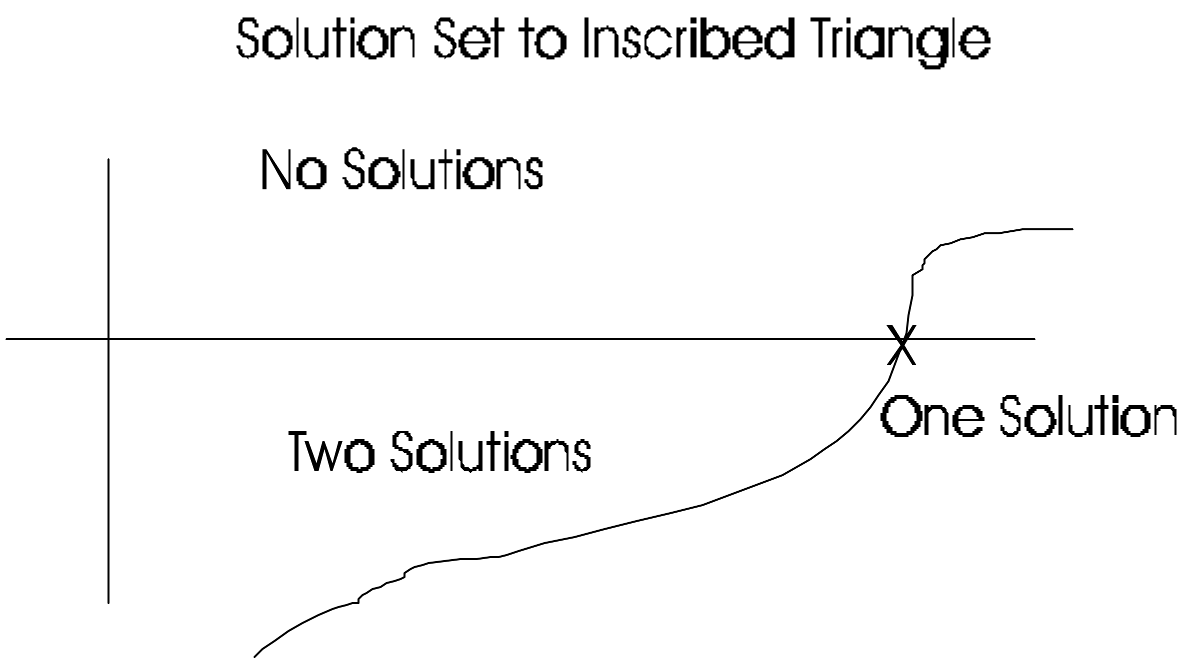
The Second Problem of the *Meno* attacks the second claim of the nominalist paradox, that no universal Ideas can possibly apply to the world of becoming. This problem attempts to illustrate how such a relationship between the Divine and the finite can be understood. It goes beyond the negative, purgative role to show in more precise terms just how two polar, contrary and “incommensurable” forms - human knowing as the rectilinear and Divine Knowledge as the curvilinear - may yet be brought together within a single unity - the limiting conditions of the paradigm triangle. The mathematical example not only demonstrates that these contrary conditions can share instantiations, but also shows us how to "measure" the conditions that determine such interactions. In the making of such "measured" constructions we are not just demonstrating the possibility of knowledge but are acting within that knowledge.

Possible human activity from a soul is captured by the rectilinear function (hyperbola). Wisdom, as the condition of making decisions based on knowledge, is represented by the perfect circle. The premise that “nothing in the human soul unaccompanied by wisdom is beneficial”, frequently proved in the early dialogues, amounts to the mathematical condition that it is only the *intersection* (accompaniment) between the hyperbola (actions within the human soul) and the circle (wisdom) which determines the beneficial, or the good, or virtue.

Our construction of this problem defeats the nominalist claim for exclusive and exhaustive alternatives to knowing in two ways. First, we have shown that the construction supports the argumentation that the one horn of the *bi-conditional* form of our First Hypothesis, that " if virtue is knowledge then it is teachable," is not true. There is an "incommensurable" area between that of the equilateral triangle and the circle, which represents that Divine Knowledge which is not "teachable". Since defeating one horn of our bi-conditional means that the biconditional doesn't hold, it also implies that the original framing of the hypothesis as an exclusive and exhaustive contrary is also incorrect.

But the construction also shows that the original negative framing of the conditionals as exhaustive contraries is flawed. The locus construction amounts to putting the two contrary conditions (separate locus curves) within a single construction and then demonstrating that there is some point which *must be* compatible with both. That point, where the two loci intersect but once, is the boundary, or limit point between the contrary conditions, and necessarily shared by both. The existence of this paradigm proves that the contraries are not mutually exclusive and therefore not exhaustive:

Fig. 3.6



This way of framing the construction is the equivalent of a kind of "completeness" proof for human knowledge. It demonstrates that between these two contrary conditions there must be a point of transition. That point in our problem is the paradigm equilateral triangle. The "compatibility" of solutions between the contrary conditionals destroys the skeptical claim that the conditionals must be exhaustive, so there can be no common ground. Just as the equilateral triangle represents that nature which is in some sense both divine and human, so the point of tangency of our solution hyperbola represents that it is neither fully inside nor outside the circle.

At the very point where we discover the "limits" of our possible knowledge, we equally discover the gate to the knowledge of the limitations of our ignorance. There is not only a "more or less" to our knowledge, but also a "more or less" to our ignorance. This continuity between our ignorance and knowledge is framed by the reciprocal and proportional relationships, as given by the applied and remainder rectangles, by which the limits are known.

This proof of the continuity of the kinds of knowledge is equally a type of completeness proof for Plato's model of knowledge. By relating the knowing of the world of becoming to that of Divine knowing, it illustrates that there are no absolute "gaps" for the possibilities of knowledge.

But equally the limit point does something further. While limiting the exhaustivity condition of the original paradox, it also utilizes exhaustivity in a positive reorientation. The point of intersection, as representing the optimum of virtuous activity within the human soul, also represents that limit beyond which there is no virtue. In other words, the paradigm is also that limit which makes the condition for wisdom mutually exclusive and exhaustive with the condition for virtue within the human soul. And as exhaustive we can now claim that, within the human soul, knowledge and virtue are universally predicable of each other and therefore convertible.

Once Socrates has shown that there are no sufficient conditions of virtue (courage, moderation, etc) which are not accompanied by wisdom (Second Contrarial Pair), he has demonstrated that the sufficient conditions for virtue are exhausted within the necessary condition of knowledge: ie. if virtue then knowledge. Plato has turned the exhaustivity condition against the skeptical rhetoric and shown that it is instead a discoverable principle of turning the analyses of inquiry into the syntheses of demonstration.

There is a final irony in this suggested solution. Socrates and Meno began their hypothetical inquiry with the assumption that if the First Contrarial Pair were both true and exhaustively exclusive, then knowledge and teachability, as biconditionally identical, would be universally convertible. And certainly by Meno's definitions of knowledge and teachability, both as *empeiria*, this would have been the case.

The investigation and development of our construction reveals a reversal of expectations. It is the precise instance that proves the non-exhaustiveness of the Contrarial Pair, the paradigm equilateral triangle, that equally demonstrates that there is a kind of teachability that is in fact convertible with knowledge. Our constructions has helped us to refine our definitions of knowledge and teachability to have arrived at those limiting cases that can convert - ideal human teachability can convert with Divine Knowledge.

3.1.3 Summary Review of Analysis in the *Meno*

My thesis has been that there is a hidden logos in this dialogue. It is not the erroneous proof that virtue is not knowledge because it is unteachable. It is rather an examination of the subtleties of logical form, and the ways in which mathematical procedure can guide our philosophical precision. The dialogue holds a key to the problem of convertibility in relation to logical and geometric demonstrations.

This relationship between convertibility and analysis is key to understanding the relevance of mathematical argument to that of philosophy. In mathematical demonstrations there are two distinct kinds of syllogism - analytic and synthetic.

Analytic syllogisms are sets of propositions which are connected through middle terms, like synthetic syllogisms. The difference is that analytic syllogisms assume that which they wish to prove.

Synthetic syllogisms move from the more knowable universal towards the less knowable phenomena. In a "scientific demonstration" an analytic syllogism is followed up by its "converted" synthetic syllogism.

Mathematical analyses are "always or mostly" convertible, and that is why they may be translated directly into synthetic deductions (Aristotle, *Post.Anal.*78a5). Philosophical analyses, on the other hand, because a true conclusion can be deduced from false premises, are most often not convertible. Aristotle holds that the advantage in the conversion of mathematical analyses has to do with the fact that mathematicians take as their premises, not accidental attributes, but definitions.

It should be clarified at this point that we have been referring to convertibility in two fundamentally distinct senses. Individual propositions may convert and arguments may also be said to convert. With propositions, conversion is just the exchanging of the subject and predicate. This kind of exchange is always valid with two kinds of propositions: particular positive statements (Some S are P/ Some P are S) and negative universal statements (No S are P/ No P are S). In order for positive universal statements to be convertible they must be biconditional universals (All S are P and All P are S).

It is often assumed (Robinson) that for syllogisms to convert the minor premise with the conclusion, as in mathematical demonstrations, all of the propositions must be biconditional or convertible. This is certainly the case in arithmetic proofs, where each proposition is an equation.

We have seen in the *Meno*, Plato is working with a more traditional concept for the convertibility of arguments. The condition for the convertibility of an analysis into a synthesis - the exchanging of the minor premise and the conclusion - is the biconditionality, or conversion of the major premise:

All P is M	All M is P
All S is P	All S is M
All S is M	All S is P

In analysis we assume what we wish to prove; for example, "All S is P" in the syllogism on the left. In synthesis we prove that hypothesis from what we previously concluded, are in the syllogism on the right. The possibility of converting an analysis into a synthesis hinges on the convertibility of the major premise.

This theme of the convertibility of analysis into synthesis is also closely tied to the larger theme of the *Meno*, the possibility of knowledge through learning. Meno's paradox disputes the possibility of learning. Socrates needs to show that some knowledge is learned.

What is more significant from the perspective of testing our mathematical method is that it is these same necessary conditions which determine that “which is not learned” - those triangles that do not fit within the circle - which simultaneously determine the sufficient conditions of that “which is learned” - those that do fit. In other words, just being able to state precisely those limits which determine the bounds of our knowledge already extends that knowledge beyond those bounds. The rectangle on the remainder is "similar" to that which is within the bounds of knowledge. And if there is continuity, then there is some limiting point that determines this difference. The mathematics gives us the tools to determine the conditions for finding such limit points.

This complementarity between analysis and demonstration is what first motivated the contention between Cornford and Robinson as to the fundamental nature of analysis. Each found some support in the writings of Pappus for the kind of analysis they supported - deductive, for Robinson, non-deductive for Cornford. We have observed that there are good grounds for asserting that there are two distinct forms of analysis.

What is at stake in this debate is more than merely historical accuracy. Whether or not there are objective, identifiable grounds to determine when or if an analysis can convert to a synthesis is the single condition which decides whether there is a “science” of inquiry. If we can turn an analysis, which increases or amplifies our knowledge, into a valid deductive synthesis, then we have grounded inquiry in a system of determinate discovery. If there is no determinate procedure, then inquiry just is an intuitive, subjective process. However, if there is some rigorous standard by which to judge the convertibility of an analytic syllogism, then inquiry can become the systematic tool for philosophy that it has always been for mathematics.

This understanding of how locus constructions, framed as diorisms, can guide our determinate use of convertibility with regard to universal propositions, can help explain another seeming confusion in the ancient "logic". In Chapter 4 of Book I of the *Posterior Analytics* Aristotle lays out what seems to be an extremely severe set of qualifications for a syllogism to be considered a *scientific demonstration*. Among these requirements is that the properties predicated of the subject in the premises, must be so “universally” or “essentially” and “to every instance of the subject”. The meaning of these strict requirements is that the subject and predicate of the major premise of a scientific syllogism must be convertible.

Let us certify:

All humans are rational
All Athenians are human
All Athenians are rational

It is not enough that the middle term is merely *contained* within the major term (the conditions for logical validity), for this to be a *scientific demonstration*, the middle and major terms must have *identical* extensions and also have "essential" connections.

Now some have taken Aristotle at his word and interpreted this to mean that scientific deductions are an extremely limited form of syllogism. My interpretation of the *Meno* provides a different way to understand Aristotle. The phrase *scientific demonstration* could mean something broader than just the synthetic or deductive argument. As in geometry, the demonstration could include *both* the analysis and the synthesis. In this reading we can understand Aristotle's strict requirement of the convertibility of the major premise.

Let us take the previous syllogism as an example. Let us say that we wanted to prove that all Athenians were rational. We would set up an analytic syllogism with this proposition as our minor premise. Then it is only a matter of finding a major premise with a subject term which is convertible with the middle term (rational):

All rational beings are ______
All Athenians are rational
All Athenians are _______

Once we have uncovered the term, "human" as being universally convertible with rational being, we can apply the rule of syllogistic conversion and exchange the conclusion and minor premise as we convert the major premise:

All rational beings are human		All humans are rational
All Athenians are rational	converts to:	All Athenians are human
All Athenians are human		All Athenians are rational

It is only under such circumstances - the biconditionality of the major premise - that the analysis will convert with the synthetic deduction.

More specifically in the *Meno*, we are to analyze the hypothesis, "virtue is teachable" in the following analytic syllogism:

The teachable is knowledge
<u>Virtue is teachable</u>
Virtue is knowledge

The major premise is accepted without argument and the conclusion is proven by the arguments in the second contrarial pair. By the rules of syllogistic conversion, I can switch the minor premise and the conclusion if the major is convertible:

Knowledge is teachable
<u>Virtue is knowledge</u>
Virtue is teachable

Now this conversion is dependent on the teachable and knowledge being convertible and therefore having the same "scope". But in our diagram it was clear that knowledge, as the Divine Circle, had a greater scope than the teachable. In order to identify teachability and knowledge, we would have to be able to re-interpret the equilateral triangle as being a paradigm for human knowing. This would be consistent with its role as representing virtuous activity as that which is both limited by and imitates the Divine.

It is important to recognize that at this point in the analysis, we are no longer dealing with a strictly syntactic procedure. The role of the paradigm in determining the convertibility of two terms with distinct scopes requires that we make a semantic move in interpreting the meaning of these ideal terms. This semantic move deepens the significance of analytic procedure towards an epistemological direction.

The *Meno* passage illustrates Aristotle's admonition on *the limitations of the analytic method*: If a true conclusion could not be proved from false premises, analysis would be easy, for then true premises and true conclusions would of necessity be convertible (An. Post., 78a5). He continues by noting that exception to this principle which finally illuminates how to incorporate its very limitations: conversion takes place more often in the mathematical sciences since mathematicians take as premises not any attributes (and in this mathematicians differ from dialecticians), but definitions (An. Post. 78a10). Scientific demonstrations, as prescribed by the dictates of locus convertibility, just are those syllogisms derived from the analytic search for middle terms in the framing of definitions.

In loci proofs, the convertibility of the premises is established by the major and middle terms being brought under a single concept within which they are co-extensive. So that finding a term which is convertible with the major is exactly the process by which one finds a middle term for a demonstration.

If I know that humans are mammals but want to be able to demonstrate the reason why, I need to find the cause or middle term. If I can find a term which is universally convertible with mammal, such as being warm blooded, I can produce the demonstrative syllogism showing that humans are mammal because they are warm blooded. It is locus diorisms which help establish the conditions under which such convertibility may come about.

So, contrary to those voices which claim that for mathematical proofs to be convertible, each step must be biconditional, Plato has shown, with his mathematical analysis that it is only the major premise which needs to be so. And it is to diorismic reasoning he turns to help make such determinations.

Congruence proofs increase or amplify our knowledge of particulars. Locus proofs relate different universals to each other and bring particulars within their range. The very fact that these two kinds of problems can work together defies the skeptical claim that amplified knowledge can't be certain and certain knowledge can only be empty.

We can now plumb the significance of another comment by Proclus on locus proofs – that they are almost always solved using indirect methods[9]. Although virtue may be a kind of knowledge which is not directly teachable, that does not mean it is fully inaccessible to human knowing. As in the finding of the diagonal which is "inexpressible", we have now "located" the precise set of relationships between teachability, Knowledge and Virtue. And we are to understand that this knowledge, like the incommensurable diagonal, is a "paradigm" which can be indirectly located, if not explicitly defined:

> It was then to have a model (*paradigm*), I said, that we were seeking the nature of justice itself, and of the completely just man, if he should exist, and what kind of man he would be if he did, and so with injustice and the most unjust man. Our purpose was, with these models before us, to see how they turned out as regards happiness and its opposite (*Republic* 472c).

The knowledge of virtue, even if it cannot be directly taught to our offspring, can somehow be captured in the nature of how a paradigm sets "limits" to our actions. In the philosophical arguments it is shown that virtue may be an unteachable knowledge. With the mathematical demonstrations Socrates has ironically shown us just how such "unteachable" knowledge might be taught: "Then do not compel me to show that the things we have described in theory can exist precisely in practice. If we are able to discover how the administration of a city can come closest to our theories, we shall say that we have found that those things are possible which you told us to prove so (*Republic* 473b)?"

The locus diorisms do not *prove* with absolute necessity that virtue is knowledge. A diorism only can demonstrate the conditions for the possibility of some hypothesis. In the Platonic rhetorical scheme it is only when one can show that an hypothesis is both possible (diorism) and Good (porism) that the hypothesis attains the status of an objective principle with intelligible necessity.

[9] Prolcus, *Euclid*, *p.166*,

CHAPTER 4

DETERMINING THE FUNCTIONAL DIRECTION OF ANALYSIS

4.1 Intelligence as Cause

In Chapter 1, I presented three issues about which commentators have debated Plato's Method of Hypothesis, or as I have attempted to portray it, the philosophical adaptation of Mathematical Analysis.

The first, based on categorizing the logical structure by which propositions are connected to the hypothesis, attempted to determine what analysis was by deciding whether it was deductive (Robinson) or non-deductive (Cornford). In Chapter 3 I considered these two different views of what analysis might be, by examining the property of convertibility as it relates to the procedures of analysis and synthesis in the *Meno*. I found that there are at least two distinct kinds of analysis, locus (deductive) and rectilinear (non-deductive), but that they do not strictly conform to the lines of disagreement between Cornford and Robinson. The deductive structure of diorismic analysis was not completely convertible in either direction as an equation is. Instead diorisms can function as a kind of "proof procedure" to determine just when an "upward" analysis can be converted into a "downward" synthesis.

The second issue in understanding how hypotheses are used in the method of analysis was to evaluate the directional or *functional* aspect of the analytic procedure. This approach, suggested by Dorter, sought to understand the analysis of hypotheses as working towards some formulation of the Good. Since the Good is that which is beyond Being and the Forms, and dialectic is the method ascribed specifically to exploring the forms (Divided Line), dialectic alone cannot seem to approach it. The analytic method must be capable of locating or determining the hypothetical Good itself. Only by hypothesizing this further unity beyond the Forms can we begin to understand the Forms themselves.

In this chapter I will attempt to show that Plato, in addition to the two previously examined techniques, develops a third form of philosophical analysis which relies on this sense of a functional direction. This procedure is an application of the analytic technique of porisms, which we developed in Chapter 2. I tried to show there that poristic reasoning is precisely that kind of thinking which relies on the hypothesis or assumption of some higher being in order to determine further what that being must be like. This kind of analysis is the *finding* of a greater field or unity within which a *functional* directionality may be determined. Since in the *Phaedo* Plato first raises the issue of directionality with regards to the method of hypothesis, I will try to show that by a philosophical application of the technique of "porisms" we can make sense of the complex argument in that dialogue.

In the process of demonstrating that porisms establish a functional or directional matrix within which to understand analysis, I will also attempt to resolve the contention of our third issue with this method. Kahn tried to establish that analysis was just a set of tools to be applied to various kinds of content or subject matter. I will make the case that he is substantially correct in recognizing distinct tools of analysis with specific fields of application. I will argue further that he has, however, overlooked the systematic aspects of the method that both unify the techniques *as* analysis and help to determine which techniques are most appropriate for which problems.

4.2 Poristic Analysis of Hypotheses in the *Phaedo*

The method of hypothesis first arises in the *Phaedo*, as a second best method to understand how “mind” or “intelligence” might be a cause in nature (99a-100c). Socrates has come to this turning point in the dialogue because of the failure of his arguments to convince Cebes and Simmias that the soul is immortal. The hypothetical explanation of generation and corruption is put forward as a response to Cebes' metaphor that the soul is like a weaver of cloaks: He may outlive many of his creations, but one will outlive him. The soul may be long-lived, but it could still be mortal. The story of generation in nature is supposed to defuse this “materialistic” objection of Socrates’ young interlocutor. Socrates must demonstrate that there is something about the nature of soul that is necessarily eternal.

There are good indications that Plato will be utilizing a poristic approach right at the start of this "second sailing" (85d). Socrates tells his two companions that they should not become "misologues" because they have at times been taken in by deceptive arguments. It is the case with arguments, as with men, "that the very good and the very wicked are both quite rare, and that most men are between those extremes" (*Phaedo* 90a). He immediately continues this warning against extremes in comparing the tall and the short, the swift and the slow, the beautiful and the ugly and the white and the black. In all cases the extremes are rare and those in between are "many and plentiful." If Socrates is to keep his companions from becoming misologues, he will need to show them how to find the middle ground in a dispute. Finding such "middles" has the character of a porism (Chapter Two, p. 66).

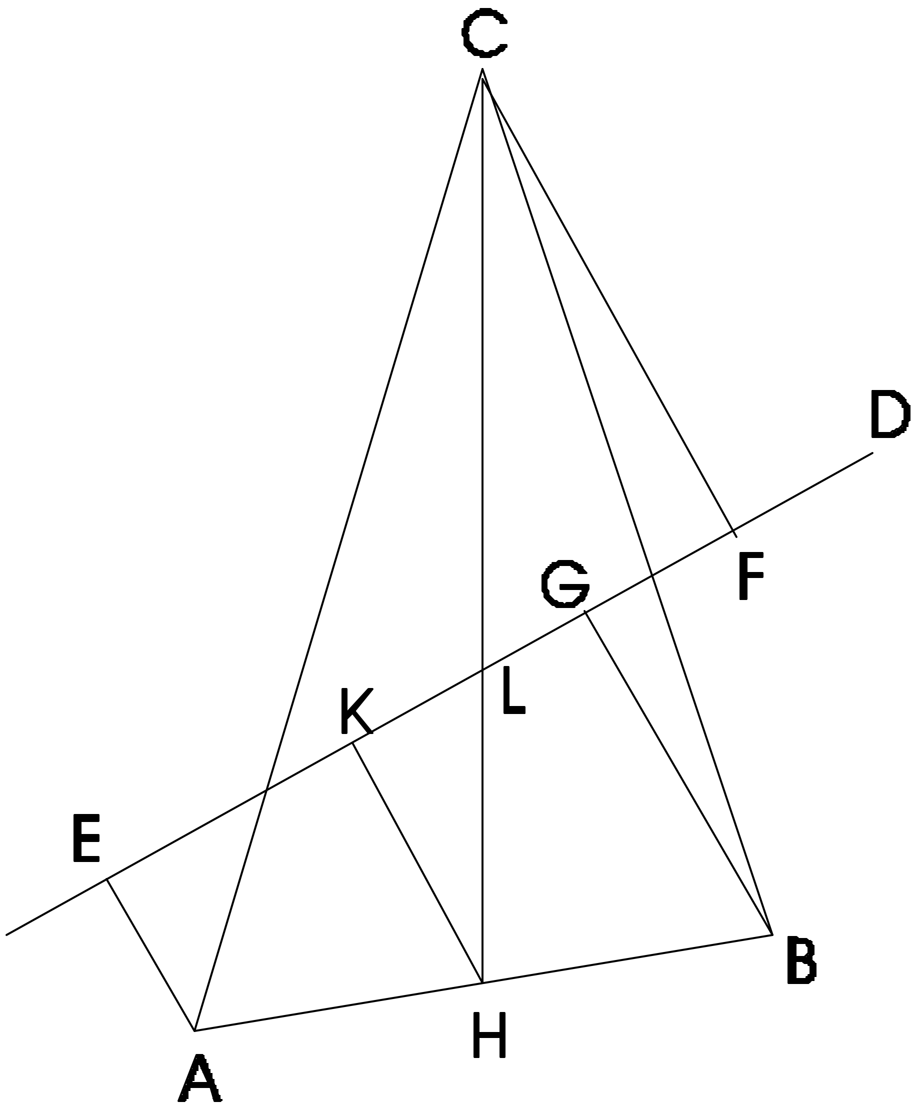
It will be helpful to recall this philosophical use of porism in direct comparison with one of its mathematical counterparts. The first porism that we examined in Chapter 2, Problem II, Figure 7, was stated as (Fig. 7):

> From any given set of points (A,B,C), a line may be drawn from a given point (D) such that [the sum of] all of the perpendiculars drawn from the points to the line on one side of the line, will be equal to [the sum of] the perpendiculars drawn to the line on the other side.

From our poristic examination in Chapter 2 we know that, rather than there always being just one solution to this problem (DE), there can sometimes be an indefinite number of solutions. Some points will give us a single solution and others multiple solutions. It is the trick of allowing the multiple solutions to go to an unlimited or indefinite set of solutions that illustrates how interesting porisms can be. It is at the point (L) where the number of solutions becomes unlimited - where finding a single determinate solution *fails* - that a deeper property of the problem is revealed. By becoming indefinite, this point discloses a higher level of determination. This point, the centroid, determines the richer solution of the porism in that any line drawn through that point solves the problem.

From this example and the ones like it in Chapter Two, we can, along with Playfair[1], draw three distinguishing characteristics about the nature of porisms. First, porisms are the "finding" of where certain kinds of problems become completely "indefinite" or develop an unlimited number of solutions. At this point of "failure" we set all of the indefinite solutions in balance with each other and determine that point which is their unity. This "center point" is a second common feature frequently associated with porisms. The third characteristic is the recognition that the existence of this balancing

Fig. 7



[1] Playfair, p. 202.

point raises the necessity that there is some higher principle that explains or cause the unification of the locus. This higher principle is the eliminating of the many indefinite hypotheses in favor of their single unifying principle.

In terms of the causal argument of the dialogue we may view each of the individual perpendiculars in the original diagram as "effects" or necessary conditions. They are the "sinews and ligaments" by which the physicalists would like to explain Socrates' presence in his cell. The indefinite set of solution lines (hypotheses) that we determine in our initial step represents all the numerous sufficient conditions that can "explain" or unify the given effects. It is the centroid point, however, that is the "cause" in the most proper sense, in that it is that unity which determines the sufficiency of all of the other solution lines. In the finding of the centroid point our "hypotheses," the lines we were supposed to be seeking, are surrendered.

There is another reason why porisms are specifically appropriate to the *Phaedo*. The *Phaedo* is a particularly difficult dialogue because it lies at the threshold between two profoundly complicated projects. It is not merely a dialogue about epistemic method and inquiry. It also speculates as to the cause and "being" of that which is under investigation. It is as ontological as it is epistemological.

Porisms allow us to introduce this further element into our method of inquiry. In mathematics "finding" the centroid may be the final goal of the inquiry. In philosophy, however, we must interpret the significance of that point and the unity that it represents. The centroid point represents the unity of the field of original lines. It can only be caused by some being that is prior to and more "powerful" than the field of lines itself.

That being or "power"(*dunamus)* is the hypothesized Good that is the cause of the unified locus.[2]

This ontological dimension of porisms becomes more evident in Socrates' rebuttal of Simmias' argument for the mortality of the soul. Socrates utilizes a poristic argument to refute Simmias' suggestion that the soul is a harmony. Socrates cites cases in which the soul opposes the needs of the body: " I mean for example, that when the body is hot and thirsty the soul draws him to the opposite, to not drinking; when the body is hungry, to not eating, and we see a thousand other examples of the soul opposing the affectations

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[2] There is a seemingly paradoxical consequence to our poristic solution. The representation of the cause as a point, indicates a lower dimension then the sufficient conditions for which it serves as their unity. Yet ontically, the cause how more "power", as that which is the reason and determiner of those conditions.

This paradox is intrinsic to the difficulty of representing higher dimensions. The dimension by which I normally think or represent the world is the "richest" that I can envision. To formulate a "higher" dimension, I must frame it negatively. I must take something away from the normal representation - unchanging, indestructible, etc. I make the higher dimension "thinner" or abstract to conceptualize it. But in "reality", this higher dimension is "thicker" than the reality I normally represent. It is normal reality "plus" some extra dimension. I believe that this view of dimension or "power" (*dunamus*) is how Plato sees the Being of the Forms (*Sophist*).

The point which the porism locates in a "center of gravity" problem is of a lower dimensionality than the line segments it centers, but also represents a greater dimensionality in the "field" or locus for which it is the center. It is this inversion between using dimensionality as a representation of abstraction and having that same abstraction be representative of a "richer" dimensionality that makes the confusion between abstraction and projection so prevalent.

This dimensional duplicity occurs with the "center of gravity" point of the porisms. The point has a lower dimensionality than the perpendicular lines for which it is the center, yet it also represents the locus which is their field. In other words, the determination of the centroid point, as an absolute unity to the field of given points or perpendiculars, indicates the existence of some prior unity. And that unity can only be ontically explained as a higher "power" or cause. So utilizing poristic to find a solution point is in effect the search for that higher dimensionality that can account for the unity that the point represents.

Our interpretation of power, with its "double" reference for the nature of geometrical points, can also help us to clear up another difficulty in interpreting Plato - whether Forms are abstract universals or “projections.” Although Plato sometimes utilizes speech which seems abstractive when referring to the nature of the Forms (*Phaedo* 100d), at other times he seems to signify that the Forms and Being are in some way richer and more robust than the world which imitates them (*Sophist* 249a).

The projective theory holds that abstractive talk about the Forms is due to the limitations of language. To represent the unchanging qualities of Being we must utilize "negative" language in comparing the Forms to the world or becoming. Being is unchanging and indestructible and immaterial. The negative reference is a "safe" way of speaking which seems to give Being a more "ethereal" nature than the world itself.

But Being is not frail or abstract on the Platonic model. It is rich, complex and *robust*. Ideal Being, as represented by the Intelligible Living Being both breathes and reasons. The immutability of this greater being is in its "extra-dimensionality" of being beyond the evolutionary flux of the world of becoming. Projection is the attempt to take the "safe" reference of Forms as abstractions and then "project" of the body (94c)." If the soul were merely a harmony of the elements, it would follow them, not direct them. If the soul is truly the ruler of the body then it cannot be controlled by the elements of the body.

The soul must oppose the diverse affections of the body: "directing all their ways, inflicting harsh and painful punishment on them, at times in physical culture and medicine, at other times more gently by threats and exhortations, holding converse with desires and passions and fears as if it were one thing talking to a different one (94d)." The soul as ruler cannot be the harmony or balance of elements, but must rather be that which balances and directs the harmony.

The poristic dimension of this argument emerges in the ontological "discovery" that Socrates draws from these observations. In ruling and opposing the "desires and passions and fears", the soul is "as if it were one thing talking to a different one (94d)," as he quotes Homer describing Odysseus striking his own breast and rebuking his heart saying,

"Endure, my heart, you have endured worse than this (94d). "

Here it is the possibility of opposition within the soul that implies the further identification of another level of being or "consciousness" outside or above the passions themselves. It is this further unity of the soul that is the eliminating of the hypothetical conditions in the final step of the porism.

The irony of this rebuttal of Simmias' suggestion that the soul is a harmony is that in the end it truly does become an harmony. The active agency of soul is able to bring the elements of soul together into a unity. At this point soul as agent and soul as patient

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it, through the phenomena themselves, onto the higher being which is their cause.

are in concord. Porisitc has somehow allowed us to make a distinction within soul itself, while at the same time, abrogating it through its own unifying activity.

In this example of poristic analysis there are two identifying characteristics of porisms that seem to recur from some of our mathematical examples. The first is that there is the finding of some balance or middle. This seems to be because the porisms that we have examined have almost exclusively dealt with "linear" loci. The goal of many of these problems is to find such balance points or lines. Whether or not all porisms have to deal with linear loci is a contested point among the mathematical commentators[3]. Certainly this kind of difficult problem is their most useful application.

Playfair obsered that in porisms the hypothesis tends to "disappear."[4] We can expect the same tto hold for philosophical porisms. Philosophically this has interesting implications for Plato's injunction that dialectic must work towards the elimination of hypotheses (*Republic* 511c).

It seems as if the two characteristics work together in anticipation of a kind of algebraic thought. The hypotheses are like the "x" or temporary unknown of a formula. When we can balance or "equate" two propositional formulae, both using the same hypothesis or "x", we then have a basis for eliminating that hypothesis.

It is also interesting to note that the porism does not itself refute Simmias. The porism is an analysis. It merely shows that the unity of soul must be prior to the harmony of its elements rather than vice versa. To argue against Simmias Socrates then shows that the soul as a harmony - a harmony having been shown to be subsequent its elements - is incompatible with the theory of Recollection, since the soul must pre-exist the body.

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[3] Playfair, p.204.

Cebes' argument takes into account this consideration of the Theory of Recollection. He concedes that the soul might pre-exist the body but like the relationship between a weaver and his many creations, one of them will eventually outlive him.

It is to counter Cebes' proposal that the soul is like a weaver that Socrates enters into his explanation of generation and corruption. Socrates will proceed in four stages. First he will review the initial beliefs that he held in the physicalist explanation of nature (96a-97b). He will consider second Anaxagoras' claim that "all is caused by mind" (97b-100c). He will next proceed to a use of the theory of forms that is considered "safe" (100c-103d), though possibly less adequate, and finally he will present a proposal for the theory of forms that is "beyond" the safe way (103d-107c).

To make the case that Plato is applying a philosophical technique that approximates the mathematical procedure of porisms I will show that Socrates takes something akin to a poristic approach in dealing with each of these three sections. In each case I will show that there is a comparing and balancing of opposed relatives that points to some definite solution of a problem framed in indefinite terms. I will also try to show that in this process, the balancing of opposites is utilized to eliminate a set of originally accepted hypotheses.

I hope to show that poristic can, if not resolve, at least illuminate the philosophical problem of the immortality of the soul. This third goal will be only partially accomplished. What I am attempting to do is to persuade the reader that this interpretive model of the dialogue is correct. I will argue that it sheds light on a text that is otherwise nearly intractable.

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[4] Playfair, p.203.

Socrates begins this section, after a long silent pause, by giving a brief intellectual autobiography of himself (96a-99e). He had first accepted the theories of the physicalists. They attempted to explain how men grew by the process of eating and drinking. They also accounted for one man being larger than another "by a head", that it was the head that caused the difference in size. Or that ten was larger than eight because it was caused by the addition of two.

In this physicalist perspective, there was present the interplay of opposing indefinites: living creatures are nurtured by the putrefaction caused by heat and cold, and thinking is caused by the elements of blood, or air, or fire. The generation of numbers could also be accounted for by the opposing actions of division or addition.

But this way of thinking about cause has some difficulties. How can the cause of two be the adding of two ones? Each one by itself is only one. How can bringing the ones near each other transform them into two? How close do they need to be to make this transformation? Equally, how could two be caused by division? If the cause in one case is the bringing together of ones, how can it also the dividing apart?

In some sense these inconsistencies are really not problematic. When I attribute Simmias being taller than Socrates by a head, there is no possibility that I mean that a head is the cause of the difference. But this understanding also points up the weakness in this way of speaking. The physicalist is attempting to explain the whole through the combining and separating of the parts. But the parts cannot explain the whole. I cannot generate the line by putting points together. The whole is of an entirely different nature and needs a distinct principle from that of the parts.

What is equally bad, I cannot explain how wholes can even have parts. When I divide a line into sections, each section is itself just another line. I need some distinct unifying principle if I am to make sense of the way in which the parts and their whole are related.

It was to find such a unifying principle that Plato turned to the philosophy of Anaxagoras, that mind caused all. He believed that having a single causal power would bring together and order the many unlimited relatives that predominated the physicalist way of looking at generation and corruption.

Socrates was initially "delighted" by this solution: "I thought that if this were so, the directing Mind would direct everything and arrange each thin g in the way that is best (97c)." Inquiring about the nature of things would only involve investigating "what is best". This solution was "after his own heart" and could explain the necessity of everything by merely accounting for the way in which it was better.

His hope with this approach was "dashed", however, when on reading further he discovered that it made no use of mind at all. Rather than truly explaining how the universe worked by a principle of mind, Anaxagoras merely gave “mind-like” attributes to the four elements and then explained nature by their *mechanical* interactions. Socrates' imprisonment would not be a matter of his "bones and sinews"(parts), but rather of his "desires and passions and fears (94d)."

In our mathematical problem, a physicalist would explain the unity of lines just by the given perpendicular lines themselves. Anaxagoras would have explained the unity of the given lines through the discovery of the "balance" lines that were hypothesized. These lines are akin to the sufficient conditions or "mental elements". Anaxagoras' solution utilized the parts of mind rather than the parts of bodies, but still lacked the singular unifying point of explanation that Socrates was seeking. To complete our poristic analogy, we would then need to go further and find that single point through which any of the lines drawn would be a sufficient reason. That point would be the locator of our "cause." That final cause of all the sufficient reasons can only be the Good, whose being has a greater "power" than any of the reasons or conditions.

It is interesting to wonder how Socrates could have envisioned mind resolving the problem of unlimited relatives in the first place. Mind is of a totally different kind than the elements of flux and becoming in the world. How is it that mind could have been accounted the cause of that to which it could have no connection? Just positing mind as the cause of becoming seems to be of little use. The Good explains everything, yet in its absolute generality explains nothing.

Socrates suggests the method of hypothesis in general, and the theory of forms in particular, as a way to see how mind can be active and determinate in the world. It is not enough for Socrates that there are discrete elements of mind that follow their character (earth, air fire, and water); the forms must show how mind requires a principle of the Good towards which its actions can be aimed. It is only by such a teleological principle that intelligence can be recognized. If we are to understand how the analytic method works to develop this hypothesis systematically, then we must come to some agreement on just how the theory of forms makes clear the intelligent influence of mind as a causal agency in nature.

Socrates attempts to make better sense of the position of mind as cause with an example of the use of the newly hypothesized theory of forms – that of the tallness and the shortness of Simmias. In examining the “safe” way of speaking of participation, Socrates states that Simmias is both tall and short in that he is taller than Socrates and shorter than Phaedo. He is tall in one instance because he participates in Tallness itself and short in another because he participates in Shortness itself.

How can he be tall because of Tallness and short because of Shortness, when Tallness and Shortness can in no way tolerate being present together (*Phaedo* 102d)? Clearly we are nearing the realm of the misology. Argument may tell us that Tallness and Shortness can in no way be present together, yet clearly most people are neither absolutely tall nor absolutely short. Experience seems to confirm the illogic of the argument.

It is difficult to comprehend fully what Plato is getting at with this example of "relatives." Tallness and Shortness fall under the Pythagorean category of the unlimited (*Philebus* 24a-25a). Forms are not of the unlimited, but rather strictly expressive of the limited. Why does Socrates introduce relatives as examples of forms?

The manner that the "safe" way implements the theory of forms is interesting for the way in which it overcomes the lack of unity within the approach of the physicalists. Simmias' being taller than Socrates is attributed to his participating in Tallness itself to a greater degree than does Socrates. On this model, both tallness and its lack, shortness, could be accounted for by Tallness itself. All sizes are but differences of *degree* of Tallness itself.

This solution is suspicious for a number of reasons. First, rather than really distinguishing between the tall and the short, this approach seems to muddle them together on a single continuum. If the only difference between the tall and the sort is a matter of degree then there really is no difference in kind. Second, the form of Shortness could have been used just as effectively to develop a scale of degrees for size. There doesn't appear to be any significant difference to determine which form we happen to choose.

But it is porisms that are appropriate to resolve these equivocal usages among terms - either Shortness of Tallness can explain Socrates' height. If two distinct forms can account for the same phenomenon, then neither can be adequately accounted as the "cause". Rather, like the centering lines in our mathematical porism, they are only sufficient conditions to explain the phenomenon. The Cause is that single point that unifies, and therefore gives the true sufficiency of all the other reasons.

I take this apparent difficulty to be the reason for the previous introduction of the form of the "equal"(74b), as a paradigm for the transcendent. The "equal" is a more appropriate expression of the limited, and a proper representation of a center or balance for the tall and short. The large and small, like the tall and short, are only meaningful in respect of each other. They have no essential meaning of their own. However, with the introduction of an intermediate term we can take the "measure" of the large and the small, or the tall and the short, by their excess and defect from the equal.

To this end we can perhaps see the ultimate function of our poristic analysis. Largeness and smallness, like cold and heat, or any other pair of opposing relatives, are each themselves a kind of unlimited. As in finding centers of gravity by finding the point where many indefinite continuities converge, the holding together of opposing relatives is a way of locating that equilibral point or “equality” which is the epistemic and ontic source of both poles. This "balancing" of two opposed indefinites "finds" that limit which makes the unlimited knowable.

With this understanding, we can recognize the failing of the alternatives that Socrates lays before us on the possibilities of interaction between opposing kinds. He tells us that not only can Tallness itself not abide along with Shortness itself but the “tallness in us” cannot any more tolerate its opposite: “either it flees and retreats whenever its opposite, the short, approaches, or it is destroyed by its approach (*Phaedo*102e)."

This move from "Tallness itself" to "tallness in us" is troubling. This causal explanation seems to be moving in the wrong direction. Poristic was supposed to be a method of seeking the "higher unity". With "forms in us" we seem to be moving "downwards" from the transcendent towards the world of becoming.

But in another sense this move resolves some of our previous problems. By moving down towards the world of becoming, this explanatory device can progress beyond the empty generalizations of the purely abstract forms. "Forms in us" give precise determinate conditions by which concrete actions can be explained.

Also, this moving down towards "forms in us" seems to satisfy the poristic condition of eliminating the hypotheses. The hypothesized forms of "Tallness itself" and "Shortness itself" have been replaced with their lower instantiations. In our mathematical problem this would be akin to explaining the "balance lines" with the originally given perpendicular lines.

But this downward movement is not totally destructive of the possibility of establishing a further unity. From our regular experience of the world we know that there are many situations where the "forms in us" *do* interact. There are people who are of "medium" height and substances that are merely luke warm. Experience shows us that there must be some unifying force or "power" within the soul to maintain such balances. Again argument has given us a result that contradicts experience.

Porism shows us that we should not be discouraged by such inconsistencies. On the contrary, the mathematical porism showed us that it is precisely the *failure* to generate an unequivocal solution line that establishes that there is a further definite solution with the center of gravity. The mathematical diagram supplies us with a visual diagram of the resolution of verbally incompatible propositions.

In discourse, when two propositions contradict each other, there are no further options. We must chose one or the other. With perceptual models, however, we may "correct" pseudo – contradictions. We can find a visualizeable structure by which two opposing propositions can be "held together" under our steady gaze. Otherwise, like "putting shoes on the wrong feet", you "don't hold the two signs each in line with its own perception, but like the bad archer you shoot beside the mark and miss – which is precisely what we call falsehood (*Theaetetus*, 194a)." Just this possibility of considering two conflicting situations simultaneously under a single encompassing unity reframes those propositions as contraries, in terms of that further respect under which they can be unified. Such pictures find the middles between the extremes that keep us from becoming misologues.

Rather than destruction or retreat, it is possible that the interaction between opposing unlimiteds can result in the harmonizing of "equals". Poristic reasoning, as examined in Chapter Two, would suggest that some point of balance can be established by which opposed and unlimited principles can be brought under a single mediating "rule" or limit. On this poristic model, the line of balance or the equal, would establish that there is a further principle beyond tallness and shortness from which they are both generated. The equal is a first approximation of the instantiation of mind as cause. Like mind, the equal represents the principle of a higher unity from which the relative qualities of the taller and shorter might spring.

But is the equal really any better than the taller or the shorter at approximating a form or a truly unitary principle? Equality itself seems to be but another kind of relationship, and relationships, as we have already seen, can be made to fall into indefinite relativeness with but the moving away and bringing close.

It seems precisely for this possible equivocation with equality that Socrates raises the problem of number generation. He has shown us how poristic can bring order to the understanding of relatives; now he is to examine a "form" in the more proper sense. Numbers are not mere relatives, but they may be used in relational references: six is twice three.

Socrates has to establish the independence of the form of “Twoness” from its various *causes* in generation: “Then would you not avoid saying that when one is added to one it is the addition and when it is the division that is the cause of two (*Phaedo* 97a)?”

In the generation of the number two, we can develop two equally unlimited paths – addition or division. We may know two as the sum of a single unit added to a single unit. This recursive way of looking at number generation predominates modern number theory. In this generative path number is understood as an accumulation of discrete entities.

But the Ancients were also interested in number as generated from the division of the continuum. The division of ratios is the basis of their extensive mathematics of musical harmony. "Cutting" a string in the "middle" produces a tone that is similar in kind to that produced by the full string, an octave up. Two in these different examples has very distinct meanings that are only united by the "form" of twoness itself. Poristically, twoness doesn't so much stand "above" these generating paths as "between" them, mediating the various relationships between discreteness and continuity.

Here we also have a fairly clear example of what Playfair meant by the "eliminating of the hypothesis."[5] "Two", as the form used to unite the distinct ways of generating "duality" is initially only an hypothesis. But as the complex set of relationships between the additive and divisive natures of two become more explicit, "two" becomes that set of relationships and is finally autonomous from either path of generation. The hypothesis has been eliminated.

Socrates' third examination of kinds of relatives is also his entree into his third way for looking at causality. It is his attempt to go beyond the safe way of speaking.

He proposes to examine the nature of the opposites hot and cold. The safe way of speaking attributes a body's warmth to its participating in the "hot itself". But now Socrates is attempting to move beyond the merely safe way of speaking: "If you should ask me what, coming into a body, makes it hot, my reply would not be that safe and ignorant one, that it is heat, but our present argument provides a more sophisticated answer, namely fire, and if you ask me what, on coming into a body, makes it sick, I will not say sickness but fever (105c)."

This introduction of "fire" to explain the hot seems again to disrupt our poristic direction of analysis. Fire seems not to be a form at all, but some sort of "substantial" agency of becoming. Are we not now dealing with the world of becoming that can in no way be brought into any kind of full unity?

In another sense, this introduction of the "substantial" element, fire, makes perfect sense. We are, after all, to try to find the nature of the causal structure of the world. We

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[5] Playfair, p. 202.

must at some point have to bring the formal unity into contact with the world of becoming if we are to assert an intelligible cause in the world.

What is it that we are being told when soul and its relationship to life are being compared to fire and its relationship to the cold? Are we supposed to determine that since fire and snow have opposing qualities, that we are supposed to finish the analogy and find the corresponding vehicle for death? But what is the vehicle for death? In this analogy the most obvious answer would be body, for body without soul is lifeless.

If we take this analogy seriously, the argument would seemingly answer the question of the immortality of the soul. The soul is immortal just because it is essentially different than body. Body is that whose activity is disorderly. Bodies degenerate. Soul, as the vehicle for life, is that which is not body. It is immaterial and therefore mortality is not relevant to it. Cebes argument about the weaver and his cloaks is just inappropriate.

But are we really supposed to accept that body and soul are totally contrary and "unmixible"? Then how are we to make sense of Socrates' response to Simmias that it is the soul that must rule the passions and appetites? Or how are we to understand Socrates' earlier arguments about philosophy and the preparation for death: "Because every pleasure and every pain provides, as it were, another nail to rivet the soul to the body and to weld them together (83d)." The passions and appetites are certainly affectations of the body. If the soul is totally unapproachable by the body, it cannot in any real sense "rule" it.

We are caught on the horns of a very sticky dilemma. Either soul is totally separate and opposed to body and therefore is immortal, but cannot in any robust sense "rule" the body. Or the soul "mixes" with the body and is its ruler. Then soul is either "like" the body and mortal or somehow "dualistic".

This dilemma is somewhat more interesting by virtue of how it imitates the split in the arguments of Simmias and Cebes. Simmias' argument, that the soul is an harmony, depends on the soul being made up of the constituents of body. It must be in concert with body.

Cebes' argument, that the body is like a cloak that the soul wears and discards, maintains that the soul and body are truly independent of each other. The mortality of the one has nothing to do with the mortality of the other.

Socrates' poristic argument for knowing that the soul rules the body is some help here. We know that the soul is some unity beyond the passions that make it up because the soul is capable of opposing those passions. The soul is not just the effect of the passions and appetites, but is that which mediates them. It must therefore be autonomous. The soul is that being or power prior to the passions, and yet it clearly has some direct relationship to them.

To see better how this poristic understanding of the soul helps to resolve the paradox of separation, it helps to recognize the full implications of the analogy that Socrates makes between the soul and fire. Soul and body cannot be fully equated any more than can fire and snow. Snow is a "thing". Fire is an "activity". Activities have a unique ontological situation that seems to bridge the irreconcilable natures of forms and things. Like things, activities are localized in a particular space and time. However, like forms, activities can be immutable. The orderly motion of the heavens is unchanging in its activity.

With fire we can see how the ontology of participation and the epistemic tool of poristic best fit together. Participation is an activity or "process". Intelligent or ordered activities always work to some end or *telos*. In this manner the activity works to unite the form or end with its participating subjects within the network of causal influence.

Poristic attempts to find the center that is more than just the centroid point. The balance point, once located indicates a unity that goes beyond both the point and the field of entities unified. The balance point indicates the *cause* of the unity itself. This cause must be of some higher being or "power" (*dunamus)*.

Activity can best unify these two ideas of balance and participation, because it has the possibility of containing its own cause within itself. Like an orbiting star, that while in constant motion never changes in its perfect pursuit of circularity, its cause is not so much in front of it, or "above" it, or in the center of its motion, although it is the recognition of a center of motion that lets us realize that the motion is unchanging.

There is a third reason to support this poristic interpretation of soul as activity. As we mentioned in Chapter 2, loci can be understood as a functional or operational expression of quantity (Proclus). The circular locus is the single continuous motion determined by the principle of equidistance from a point. The locating of a locus suggests the locating of a generating activity.

The most persuasive argument for distinguishing the active nature of the soul from the "thinglike" body, is Socrates' own poristic response to Simmias' claim that the soul is an harmony. In denying that the soul is the harmony of its elements he emphasizes the *active* principle behind such tuning (*Phaedo* 93a).[6]

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[6] The active model of the soul is best expressed by Socrates' complaint against the Friends of the Forms in the *Sophist 249a:* “But for heaven’s sake, are we going to be convinced that it’s true that change, life, soul, and understanding are not present in *that which wholly is*, and that it neither lives nor thinks, but stays changeless, solemn, and wholly, without any intelligence?”

To comprehend clearly the further development of this model of the soul, we must consider the rich complexities of the “mix”. Socrates is rejecting that the soul is just the mixture of opposites (the high and the low), as well as that it is a mixture of the limited and unlimited, or that it is the limited itself. Somehow the soul must be *like* the mixture, but also the *cause* of the mixture (*Philebus* 29e-30b). To support this model Socrates invokes Homer's image of Odysseus rebuking his own heart, "Do you think that when he composed this the poet thought that his soul was a harmony, a thing to be directed by the affections of the body? Did he not rather regard it as ruling over them and mastering them, itself a much more divine thing than a harmony (*Phaedo 94e)*."

But this is not so difficult to conceive. After all, Socrates argued that soul is immortal because it must be like that which it knows, and it knows the eternal forms. But it also "knows" and rules the mortal body. So somehow the soul, in its unity, also comprehends a many, although it is difficult to conceive how the "many" could in any way be "knowable".

But how do porisms help us to differentiate between a simple mixture and that which is the cause of the mixture? Soul is itself the unity of life in the body. But if the soul must be the active unifying agent of life in the body, to what can we account the continuing unity of the soul? In dealing with the plurality of the body, the soul must itself experience some element of the unlimited. There must necessarily be some further unifying presence within the soul to account for its continuance. This higher unity can be due to nothing else but mind or *nous*.

It is somewhat ironic how this last stage of interpreting forms has complicated our model of soul. In some sense, soul must be that power that accounts for the unifying of the opposites (centroid point). But in the model of the opposites of life and death, soul is also one of the opposites to be balanced, as represented by the originally given perpendicular lines. In addition, since it is soul that we were originally seeking in the first place, it has to be also accounted as the hypothetical "balance lines" that are eliminated by either the upward or downward movement of the porism. We have discovered not one soul, but three - 1.Soul as opposite body, 2. Soul as harmony with body, and 3. Soul as the cause of the harmony.

We must try to understand how this reading of the dialogue ties in to the ontological implications of our poristic method. Poristic analysis "finds" the balance point that establishes the unity of some diverse field or locus. But this point is not itself the form or cause of the field. Rather it represents that dimension or power that is the cause of the unity.

But "where" can we say that this power or cause "is"? It cannot be the field itself. To this degree we have to say that it is "outside" that which it causes. But equally outside does not mean "above" in the normal sense of the word. Soul, as the intermediate between form and matter is not "above" form. Also, outside does not have to mean "separate" in any absolute way. While the poristicly located cause is in some way outside its object, it can equally said to be centered at the point of balance. This strange, complex kind of relationship would be impossible to visualize fully were it not for our mathematical modeling of poristic analysis in Chapter 2. The poristic solution of the center of gravity simultaneously locates a higher dimension to the problem for which the solution point is merely the sign.

Within the soul, this center is that place of quiet from all influence where we can go to deliberate freely about future possibilities[7]. This unity, as we have tried to show, is that of intelligent activity within the soul. It is the "care of the soul" which allows the individual to continue to rise to that intellectual activity which is the eternal unity of soul in general. It is the recognition that it is intelligent activity that is the true nature and function of soul which first allows the individual to "let go" of his hypothesis of soul as ruler of the body and instead see soul as bodiless. This is the "turning" of the soul away from the shadows and towards the light. It is also intellectual activity that is the experience of an eternal moment for which there is no individuality. It is this identification of soul with the eternal, non-personal activity of the intellect that frees the soul long before the cessation of the body, and is the true "practice of dying" of the philosopher.

4.3 The Applicability of Analysis

Having answered our first two questions on the use and kinds of analysis, we are in better position to seek resolution of our third question: how does the way in which we examine analysis affect what we discover about the method? We have so far looked at analysis from the perspectives of logical structure (Chapter 3) and directionality (Chapter 4). We had originally raised three different approaches taken by contemporary commentators: logical structure (Robinson and Cornford), directionality (Dorter) and applicability to subject matter (Kahn).

I would now like to claim that in finally understanding this complex set of directional interactions, we have not only resolved Dorter's issues (directional), but have

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[7] We must remember that Socrates began this second part of the dialogue with a long, silent pause.

concurrently resolved our final issue in interpreting analysis - the content oriented approach of Kahn.

Kahn holds that analysis is just the full complement of distinct tools that the mathematicians utilize to solve problems. He does not want to commit to a thesis of either directionality or logical structure. Instead he holds that in different circumstances, different tools are appropriate. Understanding analysis is just a matter of knowing which tools are appropriate for what kinds of problems.

This "segmented" approach to understanding analysis raises the image of the Divided Line. Different sections of the line represent different objects of knowing and are accessed with distinct kinds of methods. If we are to make the transition from our structural and directional examinations of analysis to one of applicability to different contents, we will need to try to "fit" our model of both kinds of directionalities onto the Divided Line.

Before I attempt to follow through on this comparison, some difficulties need to be anticipated. First, it must be doubted that the Divided Line is appropriate at all as a means of examining the application of mathematical methods. The realm of mathematics is clearly limited by Plato to the third section of the Divided Line. How could mathematical method have any range beyond this one section?

I have tried to show throughout this dissertation that there are many parallels available among the structures of mathematical reasoning and that of philosophy. These parallel structures (diorisms, porisms) indicate that even though philosophy's most appropriate realm may be that of the forms, the tools of philosophy, like those of mathematics can be utilized to make sense of the world of becoming as well. To the degree that we can consider a subject matter a "thing" the analysis of wholes and parts may be appropriate. In cases where we are considering how an instance may or may not fall under a universal, dioristic analysis may be called for.

We should be cautious, nonetheless. Even if we determine that some content or subject matter is most appropriate to a certain method of analysis does not make that particular application exclusive or exhaustive. While all of these analytic techniques are originally mathematical (third section), they may be applied to other sections to the degree and respect to which those sections are similar to the representations of the third section.

In terms of this "secondary" application of our analytic techniques beyond the third section of the Line, it seems that rectilinear analysis would be the least problematic. This analysis deals exclusively with parts and wholes. It exclusively deals with "things" and their parts. Forms, after all, are perfect unities and cannot be wholes of parts. This evaluation would limit rectilinear analysis to the lowest two sections of the Divided Line.

Diorismic analysis is somewhat more difficult to interpret. It is the determination of the conditions which determine the possibility of an instantiation. Diorismic analysis is the transition from the Forms towards their instances in ordered activities. This method fits most appropriately between the fourth and third sections of the Line. The third section, the mathematicals, is transitional between things and forms. In the ontology of participation, this also makes the third section the realm of activity as participation. Mathematicals are just the way of knowing such ordered activity.

Poristic is in some ways the most difficult to locate on our Divided Line. We might be tempted to put it at dead center of the line, between the things and the mathematicals. Porisms, afterall are the "centers" or balancing points.

But this placement would be deceptively simple. Porisms, as we witnessed in both our mathematical and our philosophical examples, have two distinct movements. There is first the movement from the necessary conditions (given line segments) to the sufficient conditions (balancing lines). But then there is the finding of the absolute centroid - our second movement.

Utilizing this model we could situate the first movement (finding of the balance lines) of poristic in the boundary between the "things" and the "mathematicals." Here the mathematicals would represent the necessary conditionality of the things of becoming. Then in the second movement (finding of centroid), that from the sufficient conditions to the absolute cause, we can appropriately place this transition between the mathematicals and the forms themselves. This motion would signify that elimination of the hypotheses (balance lines) that Plato tells us is the final movement of dialectic. This second movement is the surrendering of our images (hypotheses) for the true Ideas.

This simplistic approximation of the application of our three methods of analysis can in no way be final. As we have seen in the *Phaedo,* something much like poristic analysis seems to be applied to everything from relatives (tallness and shortness) to substances (fire). Nonetheless, I believe that I have shown that Kahn is partly correct in concluding that each technique may be best suited towards a particular kind of content. The deficiency in Kahn's approach is that he does not recognize the *systematic* relationships, both structural and functional, that help determine both what is the content most appropriate for each technique and when and how it may be utilized beyond that content.

In the end, Kahn has not answered the difficult question of "what is analysis in Plato?". If analysis is an assortment of distinct tools, then it is not "one" thing. In what way can this multiple phenomenon be considered a true unity? If we are to move beyond Kahn, we must do better at bringing some single understanding to this set of techniques.

But if we are to find that single unity or definition that ties together our various sorts of analytic techniques, we must first perhaps "hypothesize" what a definition is in the first place.

The Divided Line, as well as much that is said in the dialogues, would suggest that there are distinct kinds of definitions for distinct kinds of being. In the lower two sections of the Divided Line, we are dealing with "things" that are primarily part-wholes. Setting the name in balance with the parts would seem like a sufficient definition. Part-whole definitions are *tou hoti* and locate the functional *uniqueness* of the entity with a kind of *distinctness.*

With an idea or form, such as virtue, it is the genus and differentia that gives us the proper definition. In the Second Problem of the *Meno*, the equilateral triangle locates the species within the genus (circle), while the area between the triangle and circle is most properly the differentia. Generic definitions are *tou dioti* and locate the classificational *completeness* of the instantiated class with the quality of *clearness*.

It is porisms as the middle transition point in the Divided Line that give us the definition of definition in the most proper sense. A definition, like a poristic point, is just that unity that holds its many instances as "one". It is the case that with the most complex phenomena, such as humans, there are a great deal of seemingly equivalent functional organizations and also many seemingly equivalent possible differentia. It is only when we set the differential properties such as "being able to laugh" and "having the longest intestine" in a kind of equilibrium that we can find that "point" which explains all of the functional parts and differentia together - rationality. Such "organic" definitions are maximally clear and distinct , as defining specific phenomena both completely and uniquely.

Analysis, in each of its three modes, is the seeking of the most appropriate definition for each kind of "being." In this essential function of the method analysis can be said to be the seeking of that which makes the thing analyzed what it is. Analysis is the seeking of the cause of the analyzed.

But more than this generic functionality to analysis, there is another unity by which these three procedures may be tied together. As being identified with the transitional movements between the sections of the Divided Line, as the map of all possible knowability, there is a single *telos* or *ethos* under which they all equally work. Together they supply the bridges between which parts, things, activities and ideas can interact through a common end - the unity of everything *that is in any way knowable*. The Divided Line puts all ontological distinctions within a single continuum which defies any radical duality.

CHAPTER 5

CONCLUSION

We started this dissertation with the question: What is mathematical analysis and how is it utilized in Plato's philosophical inquiries? In the initial investigation (Chapter 1) we determined that our question was intimately involved with an equally contentious set of other questions: what are dialectic and the method of hypothesis? We determined to consider philosophical analysis as the inter-working of dialectic and hypothesis and that to better understand what analysis is in Plato we would have to explore the technical details of mathematical analysis and compare them to the ways in which Plato describes hypothesis and dialectic.

We carried out this exploration utilizing three distinct approaches to understanding the relationship between dialectic and the method of hypothesis. The first approach sought to distinguish the method of hypothesis from dialectic by way of the logical structures that interconnected propositions within the arguments of the two techniques. This approach was pursued in the controversy between Cornford and Robinson.

The second approach was to compare the two methods in terms of their functional directionality. Could either of the methods be identified as either "upward" or "downward"? This approach was that followed by Dorter.

The last attempt to understand these two methods was that by Kahn. Kahn held that since the bonds analyzed by both dialectic and the method of hypothesis were presented as "ontic" constructions rather that "logical" connections, neither method could be construed in a strictly logical fashion. Instead each method needed to be applied according to its ontological "fit".

From our examination of two dialogues in which the method of hypothesis is first presented, I believe that we can draw the following conclusions.

First, due to the distinct natures of diorism and porism we can conclude that philosophical analysis includes both "downward" and "upward" movements. What is more difficult to absolutely decide is which technique moves in which direction.

Diorism is an "upward", deductive analysis that is subsequently "converted" into a "downward" synthesis. In that the analytic part of the movement is "upward" it is appropriate to consider this technique a deductive example of the method of hypothesis. This would be contrasted to the "upward" movement of hypothesis in a congruency problem, which would be non-deductive.

Porism is a more complicated process. In the argument for the active nature of the soul in the *Phaedo*, porism seems to work "upward". By finding the point of unity for a field or locus I may then postulate a higher power or being that is the cause of that unity. But in the latter examples, porism is utilized in the reverse direction. After the opposites of hot and cold are united in a comparison, the examination moves "downward" to consider the opposites fire and snow. It seems that once a unity has been established, any number of lower pairs of opposites can be hypothesized from that unity. In this process, the "upward" movement, which is the elimination of the first hypothesized opposites, seems to compare well with the description of dialectic in the *Republic*. And the "downward" movement, which is by opposites, also seems to compare well with the dialectical technique of *dieresis*. Porism seems to be doubly tied to dialectic.

I believe that our examination has also shed light on the possible ways in which dialectic and hypothesis can work together in philosophical analysis. In the *Meno* problem, a diorism, we were looking for the conditions for the possibility of a particular being subsumable under a universal. The conditions turned out to be the looking for a "mean" - the mean proportional between the two segments of the diameter. Porism is that technique which is based on finding "means" or middles. We should consider that there might be dialogues in which these two techniques are utilized in concert.

If our examination of the nature of mathematical and philosophical analysis has been adequate, it should aid us in shedding light on some of the other questions that have arisen during this investigation. First, what does the nature of mathematical analysis tell us about the nature of mathematical thinking? Is it ampliative or universal or both? Also, in what ways do philosophical analysis answer the arguments against the possibility of universal objective knowledge as put forward by the skeptics and relativists? Has Plato defended the possibility of philosophical inquiry against their attacks? And finally, what is the nature of both mathematical and philosophical inquiry? Is it an algorithmic science or a merely creative psychology?

5.1 The Logical Structure of Mathematical Analysis

In examining how Plato utilized analysis we have come to recognize three distinct kinds: rectilinear, diorismic and poristic. This recognition will help us to make some sense of the history of disagreement among commentators as to the nature of analysis.

The simplest kind of analysis is that of rectilinear figures. There are many mathematicians who would not consider this kind of problem under analysis proper. Rectilinear analysis is always definite (one unknown) and their unique contribution to the investigation of mathematical knowledge is that they are epistemically *ampliative.*

All rectilinear constructions (Slave Boy Problem), as whole-part problems, have certain features in common. When done correctly, they each transform an indeterminate problem into a determinate one. Determinate problems are those problems which follow the law of three (*Timaeus* 17a). If I am given three of four elements in a proportion, I can always determine the missing elements. Of course this rule can be extended to account for finding the missing element within any (n-1) number of given elements in a relationship.

Given an isosceles triangle, I cannot immediately know anything about its angles. There are six parts to the triangle, three sides and three angles, and I only know that two sides are equal. If I reduce the ratio of 6:2 (six parts, two known) to 3:1, it is an n-2 (3-1 = 2 unknowns) situation, which is indeterminate. If I make any arbitrary construction into parts, my situation is not improved. I construct two triangles with twelve parts. While I still have the equality of the two sides, I now also have the common side between the two part triangles. So my new informational situation is twelve potential parts with four knowns. After reduction, we see that the situation remains an n-2 indeterminate problem.

If instead we utilize our option to place a single, controlled constraint upon our construction, let us say by bisecting the angle between the two equal sides, we end up with two triangles for which we now know six pieces of information - two sets of equal sides and a set of equal angles. This reduces to the 2:1 ratio which reflects the n-1 condition of a determinate problem.

There is something else radically prolific about the way in which rectilinear constructions can *produce* information, or new knowledge. In rectilinear congruence problems there is a disjunction between the conditions for knowledge of a determinate comparison (sufficiency) and those given, necessary conditions of the comparison itself (necessity). This means that I can "know" a rectilinear figure adequately (sufficiently), before I know it completely.

To have two triangles congruent, there are six necessary conditions that must be met - the equality of all of the sides and angles. However, to determine *that* two triangles are congruent, there are three sets of partial conditions that are *sufficient*: three sides equal, two sides and the included angle, and two angles and the included side. We must make a distinction here. We can not say that there are less sufficient conditions than necessary conditions for congruency. The sufficient conditions must already somehow latently include the necessary conditions in order for them to be truly sufficient. We must make a distinction between ontic and epistemic conditions. Three conditions are epistemically sufficient for the determination of congruency between two triangles, from which six necessary and sufficient ontic conditions may be then deduced.

This “explosion” of conditions within rectilinear constructions, is not due to the projective properties of straight line constructions. These can only produce proportionality, which relies strictly on the rule of three. It is the hybridization of the projective properties of the ruler with the metric qualities of the compass which seems to produce the expansive qualities associated with the *closure* of figures. And this principle of the closure of figures is what correlates to the principle of wholeness in the part/whole relationship of rectilinear constructions.

What I have attempted to illustrate, is that rectilinear problems, by virtue of being part-whole constructions, hold the promise of revealing new relational knowledge. The logic involved in discovering their solution is the *determinate*, backward reasoning from the necessary epistemic conditions of the conclusion toward some expanded set of sufficient conditions which may then be deduced. This is typically thought of as the kind of abductive reasoning which mathematicians and detectives utilize. Its mysterious allure is the one-to-two proliferation of sufficient to necessary conditions which rules the whole-part individuating property of closure. This doubling factor, when supplemented by the increase of information derived from the auxiliary construction of parts from a whole, is the key to understanding the fundamental nature of determinate ampliative knowledge. These two factors have been under-appreciated for their contribution to the power of geometrical thinking.

Despite the ampliative nature of determinate inquiry, it is of limited use in philosophical problems. It can only apply to the analysis of whole-part problems and those situations modeled on them. It is a backward driven procedure that works from the sufficient conditions of the conclusion sought back towards the necessary conditions of what needs to be assumed. This form of analysis appears to be the process of deliberation towards action of Aristotle and the method of discovery of Cornford.

Locus analysis, as represented in *diorisms*, works within the conditions of inclusion under universal principles. Diorisms, as the type of analysis used in the second *Meno* problem, involve finding the conditions under which certain kinds of instances may be included under a more general, continuous curve (loci). It is the significance of this specific locus phenomena (diorisms) which first allowed the ancients to “apply areas” in proofs. This is the equivalent of the substitution methods of modern algebraic logic. Different rectilinear figures with the same area can be equated. Equally in reasoning, when two concepts can be brought under the same idea-locus, they are universally convertible.

Diorismic analysis, although following deductive rules and depending on the convertibility of the analytic syllogism, is still not the "algebraic" set of identity relationships of modern mathematical logic. Robinson is wrong to assume that all convertible syllogisms must reduce to this kind of representation. There are very precise conditions under which such convertibility can take place and these problems have been the subject matter of Aristotle's *Prior Analytics.*

The conditions on convertibility that the Second Problem in the *Meno* explores, amounts to the establishment of a kind of "proof procedure". By illustrating the distinction between incomplete, but real knowledge and only apparent knowledge, the diorism in the *Meno* sets the standard for when indirect knowledge may be convertible with the forms themselves.

However, the convertibility of its analytic propositions should not be construed as a “gain” in knowledge. The “reduction” which takes place in the converting of these propositions is more of a *lateral* move than one of directional gain or loss. The substituting of identities has the effect of reducing the “ontic” character of the figure to one of a different form, rectilinear to triangular, while keeping the “quantity” between the forms (area) the same. In this manner an indefinite locus problem can be reduced to one which has a whole-part or rectilinear resolution.

The differences between these two kinds of analyses (rectilinear and diorismic) is represented in a debate raging at the Academy, which, in some form, is still alive in the contention between Robinson and Cornford. Proclus informs us that upon the accession of Speusippus to the head of the Academy there was an intense debate about the nature of mathematical method. Speusippus and his followers contended that all mathematical propositions were *theoretical*, since "problems of geometry are of a different sort than mechanics."[1] Eudoxus and his student Menaechmus held that all mathematical propositions were *problematical*, since "discovery of theorems does not occur without recourse to matter."[2] This debate would continue through Medieval times right up until the dawn of modernity with the distinction between demonstrations *tou hoti* (problematic), or "of the fact", and *tou dioti* (theoretical), or "of the reasoned fact".

This difference also seems to confirm Pappus' use of the *upward* and *downward* reference in regard to the two kinds of analysis. While both inquiry types of thinking are analytic with reference to their role in the demonstration, locus proofs, as exemplified by diorisms, work *down* from universal *ideas* to their *instances*, while congruence proofs work *up* from the necessary properties of *parts* to the sufficient conditions of their *wholes*. We have seen that another kind of locus problem, the *porism*, also seems to work "upward."

The strains of this dispute are still visible in the contention between Robinson and Cornford (deductive vs. non-deductive). But there is a problem with both positions. All parties seem to agree that it is "chimeric"[3] to hold that mathematics can be both "truth preserving" (theoretical/deductive) and "ampliative" (problematic). Sides seem to have to choose between which of these two characteristics of mathematics is more essential to its nature.

Plato, by including both of these two problems in his dialogue about inquiry and the nature of human knowing, seems to resist this "modern" choice. Not only does each problem seem to fit under one or the other of these two categories for mathematics, but

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[1] Proclus, *Commentary on the First Book of Euclid's Elements* (Princeton, 1970), p. 65.
[2] Ibid.
[3] Wilbur Knorr, *The Ancient Tradition of Geometric Problems* (New York, 1993), p. 75.

each deals with one of two fundamental philosophical questions which need to be answered for this dispute to be settled.

The Slave Boy problem, one involving the laying out of the constituent parts in the auxiliary construction, is an example of reasoning "from the facts" or particulars. It is presented in order to illustrate the possibility of "learning" new knowledge through "recollection". Its formulation is to demonstrate how the *ampliative* nature of mathematical problems can elaborate the *ampliative* possibilities for learning.

But these rectilinear congruence proofs, by the nature of their specific connection to a particular part-whole relationship, are not "truth preserving" beyond the limits of those same species of figures. Their truth cannot be "translated" or "reduced" to any other truths. The universality of rectilinear congruence proofs, being derived from a material induction from a figure, is only relevant to those arguments from species which preserve the same kind of analogues with that figure. They are useful within well described physical models, but of limited use to the broad, universal relationships more typical of philosophy.

The syntheses into which rectilinear analyses are converted, since they do not derive from universally valid major propositions, cannot be said to be "truth preserving". They rely on relational propositions and, although they are "ampliative", are only demonstrations *tou hoti* or "of the fact".

The syntheses into which locus analyses are converted are "truth preserving" and *tou dioti,* or "of the reasoned fact". The model of class inclusion that is represented by the diagrams of Euler and Venn establishes that asymmetric relationship that holds among a well elaborated hierarchy of ideas. The rules of syllogistic determine within this hierarchy when some predicates can be converted with others based on the semantic content of their definitions. But any such "reduction" of one term to another will always be a "lateral" move of equating two concepts. They are never ampliative and cannot disclose new knowledge.

The overarching function of analysis is the bringing of the ampliative propositions that are attained from syllogisms "of the fact" within the rule of those universal propositions derived from syllogisms "of the reasoned fact". This process works in two directions, both downward (diorisms) and upward (determinative). In this manner Plato sees the analytical project as "weaving" the two kinds of analysis together into a method of inquiry which is capable of "discovering" that mathematical truth which is both ampliative and truth preserving.

Poristic analysis is the basis for setting disparate and discrete magnitudes in equality with each other. As such poristic is the threshold to an algebraic representation of the logical process. Poristic analysis does finally support Robinson's claim that there is a fully convertible kind of analysis and that this method is what is most commonly known as "modern" analysis. Robinson, like all of the participants in this debate, was merely negligent in not recognizing the various kinds of mathematical and philosophical techniques in play.

5.2 The Utility of Analysis

If our analysis of analysis has been at all successful, it should aid us in answering the two major questions raised about analysis in our preliminary investigation: 1) How does Plato utilize analysis to answer the skeptical and relativist claims of the empiricists and the sophists? and 2) Is analysis an algorithmic proof procedure or a psychological heuristic?

In the *Meno* Plato utilizes two distinct kinds of analysis to counter the skeptical arguments against the possibility of learning. The skeptical position, as "memorized" by Meno from his teacher Gorgias, is that we can never learn anything since we would never have to learn what we already knew and we could never be aware of what we didn't in any way know.

This paradox is based on the skeptics' dichotomizing of the kinds of knowing. There are eternal truths that we must know completely or not at all, but they are empty and not applicable to the world. And there is factual experience, that can be "learned" and increased, but is not "knowledge" at all.

The two mathematical examples of the *Meno* are each supposed to defeat one of the two skeptical horns. The Slave Boy Problem shows that "factual" or particular kinds of knowledge can lead to a kind of determinate objectivity. The whole-part construction of the Double Square is an example of the ampliative possibilities that can be "seen" in congruency proofs. This knowledge is not derived from the syntactic laws of syllogism but from the semantic interpretation of the figure and its parts. The slave "learns" certain undeniable principles from the particular diagram, even if he cannot put them into propositions.

The Second Problem is meant to show that the Ideas or forms, as represented by the circular locus, can be apprehended by the partial and indirect methods of dioristic analysis. Eternal truths can be approached or "learned" through indirect means.

The model of poristic reasoning is utilized to fight the relativism which emerges from the continuum paradoxes of Zeno and Protagoras. Within this logic, if any two qualities may be put into a comparative relationship, then there is always a “third” which may be found to relativize the relationship. And then this third may in turn also be relativized by a “fourth”, and so on in an unlimited fashion. This Third Man approach, “perspectivizes” all relationships and destroys the possibility of establishing an “objective” measure of normativity. This is the logic of “man is the measure.”

Socrates needs to get the sophists to agree to some “assumption”(hypothesis) before the analytic process can begin. In this case that assumption is the existence of at least a *subjective* or relative normativity (hypothesized balance line). The sophists need this minimal agreement if they are to “sell’ their own teachings (*Protagoras*).

Poristic reasoning takes advantage of this agreement on the existence of some norm to then lay out the conditions for its objective determination. Once there is an agreement that there is a “better and a worse”, Socrates can utilize the philosophical equivalent of the “centroid” construction to show that there is some point towards which all of these betters converge. That point, where the many indefinite “goods” become maximally indefinite, is the poristic unity from which all Good is “balanced” – the *centroid* or center of gravity.

Finally, we must determine if our examination of analysis helps us determine whether this method is either an algorithmic map, or merely a psychological guidepost. Analysis, in all of its forms, always relies on some “constructed image” or hypothesis. This requirement fundamentally transforms the interpretive demands on the inquirer. Although algorithmic-like steps may be followed to determine systematically the appropriate kinds of constructions, the kind of lawful procedure followed will always be more *regulative* than constitutive. Discovery therefore is neither merely algorithmic nor subjectively psychological. It is rather the systematic conditioning of the inquirer’s soul in the reciprocal process of ordering the world. It is “lawful” and psychological. It is *mathesis*.

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